Structural Equation Modelling

Short course in Applied Psychometrics

Peterhouse College, Cambridge, 27-29 March 2012

This course

The course is funded by the ESRC RDI and hosted by The Psychometrics Centre

Tutors

Jon Heron, PhD (Bristol) jon.heron@bristol.ac.uk

Anna Brown, PhD (Cambridge) ab936@medschl.cam.ac.uk

Tim Croudace, PhD (Cambridge) tjc39@cam.ac.uk

	Day 1	Day 2	Day 3	
9:00-	Coffee on arrival Introductions + Aims of course	Lec-6 – Special issues in CFA Correlated errors Bi-factor modelling Method factors Multi-group CFA	Lec-9 – SEM Incorporating latent traits into path models.	9:00-
9:20-				9:20-
9:40-	Lec-1 Mplus modelling framework			9:40-
10:00-				10:00-
10:20-				10:20-
10:40-				10:40-
11:00-	Coffee	Coffee	Coffee	11:00-
11:20-	Lec-2 – Regression models	Lec-7 – Path models 1 The basics / figures / Identification/ model fit/ equivalent models	Examples 5 – SEM EAS - SEM	11:20-
11:40-				11:40-
12:00-	Examples 1 EAS - regression models		Wrapping up, further reading and questions	12:00-
12:20-				12:20-
12:40-		Examples 3: SZ paper.	Lunch and depart	12:40-
13:00-	Lunch	Lunch		13:00-
13:20-				13:20-
13:40-				13:40-
14:00-	Lec-3 - CFA with continuous variables	Lec-8 – Path models 2 Model refinement Direct and indirect effects Binary mediators - logit/probit		14:00-
14:20-				14:20-
14:40-				14:40-
15:00-	Lec-4 – EFA with continuous variables			15:00-
15:20-				15:20-
15:40-				15:40-
16:00-	Coffee	Coffee		16:00-
16:20-	Lec-5 - CFA and EFA with categorical variables	Examples 4 Path model using EAS		16:20-
16:40-				16:40-
17:00-	Examples 2 EAS – CFA/EFA			17:00-
17:20-				17:20-
17:40-				17:40-

CFA + Path Analysis = SEM

So now it's time for Path Analysis

Before lunch

- Path Analysis Models [1]
  - Model specification and identification
  - Model estimation
  - Model fit
  - Equivalent models
- Examples 3 – Schizophrenia model

After lunch

- Path Analysis Models [2]
  - Model refinement (path testing)
  - Direct and Indirect effects (mediation)
  - Mediation with binary measures
  - ~~Skewed data and bootstrapping~~
- Examples 4 – Path Analysis ~EAS temperament

Path Analysis 1

The Basics

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Steps of SEM (from Kline)

1. Specify model
2. Model identified? (if no, go to 1)
3. Collect data
4. Assess model fit
5. If model fit poor then re-specify
6. If model fit good
    1. Interpret estimates
    2. Consider near equivalent models
    3. Report results

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Model specification

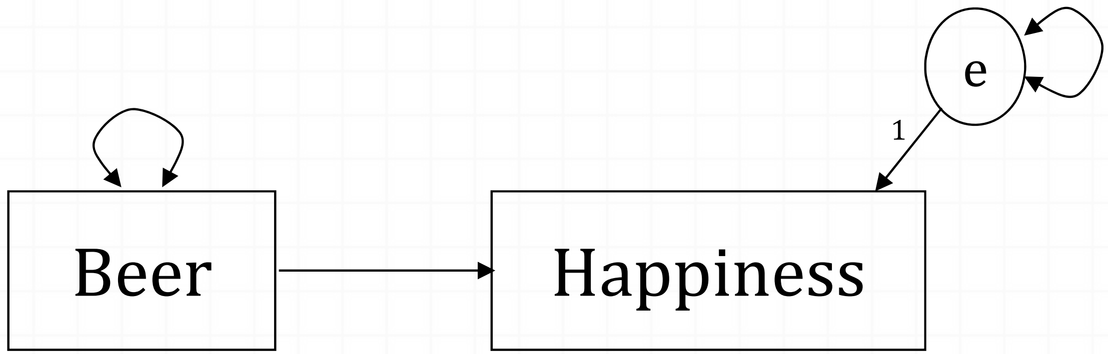
- How does THEORY say our concepts should relate to each other??
- Do this BEFORE looking at the data
- Or even better, before COLLECTING the data

- Knowing what data you have can influence your model – “ooh, how can I use my ten measures of emotional symptoms….?”

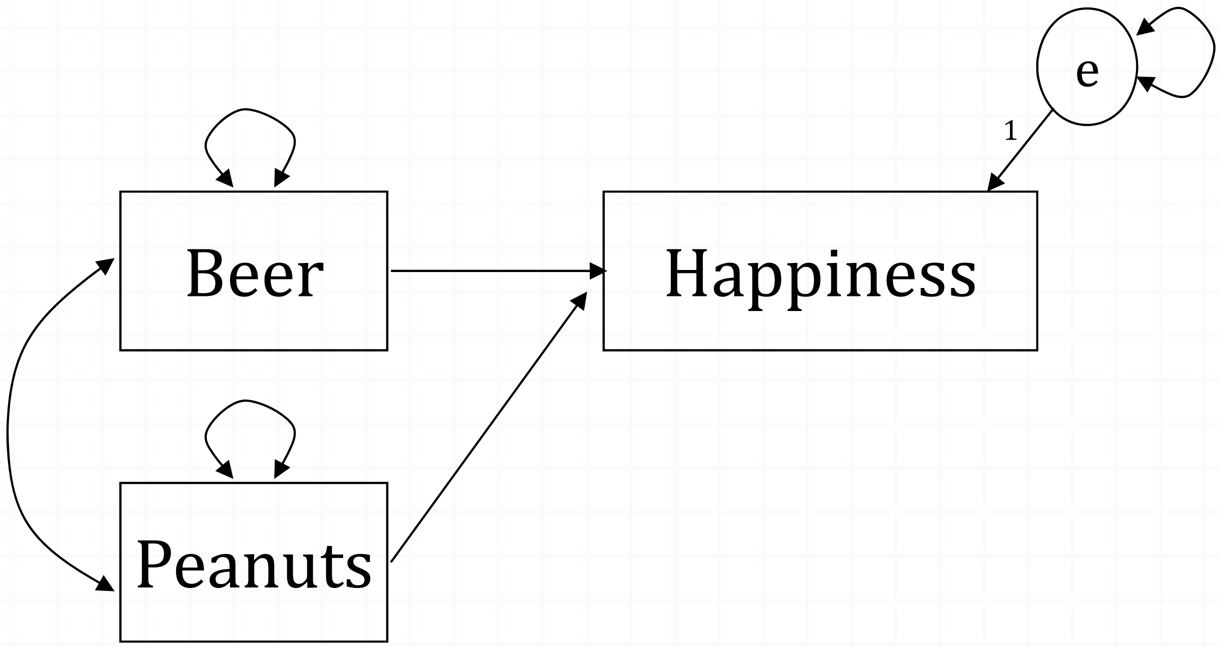
Relating stuff to other stuff

- Single / Multiple causes
- Direct / Indirect effects
- Uni- / Bi-directional effects
- Independent / correlated errors or residuals

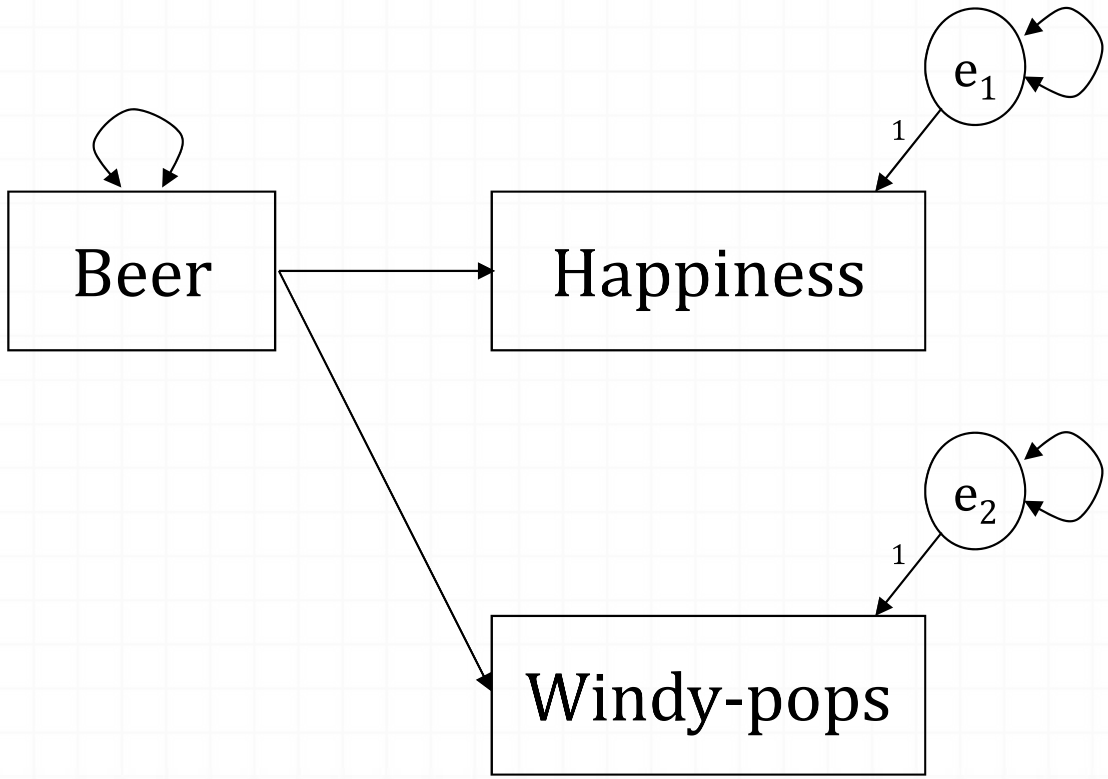
Single cause



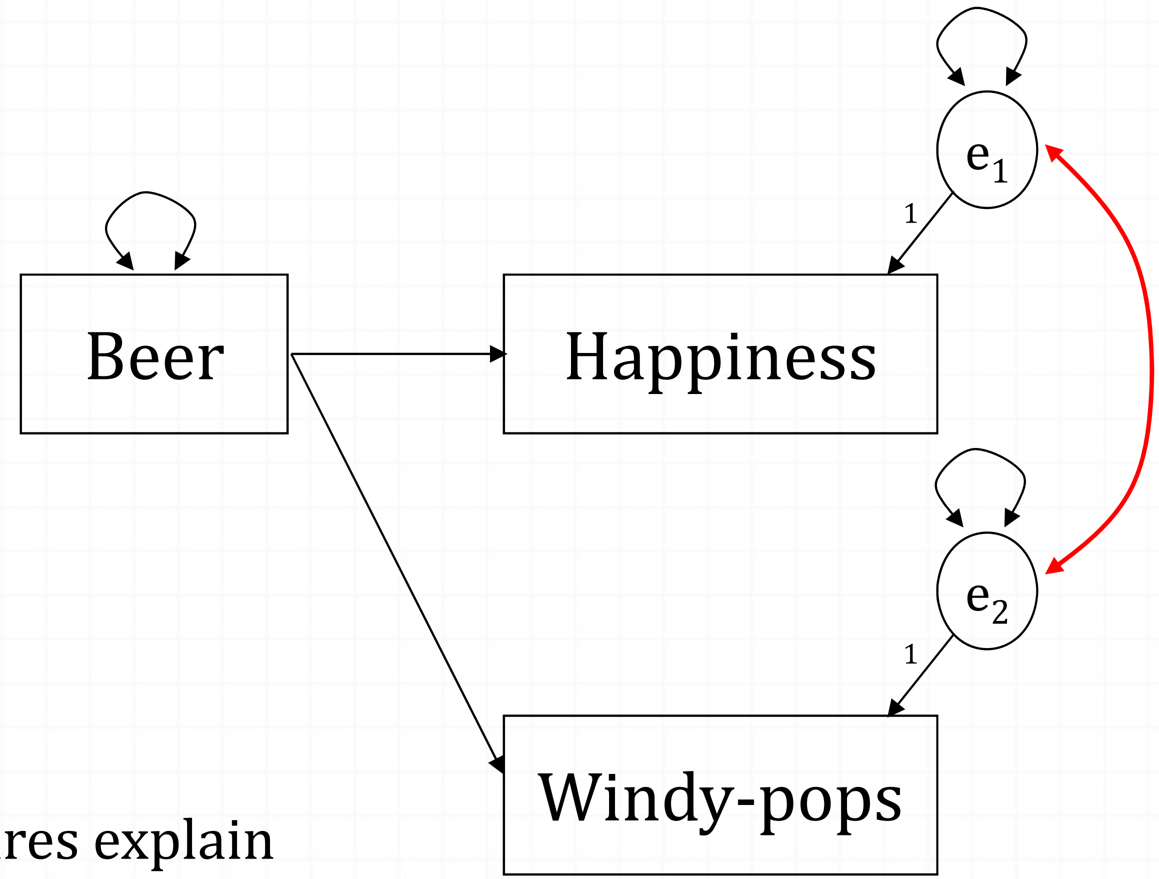
Multiple correlated causes



Multiple outcomes

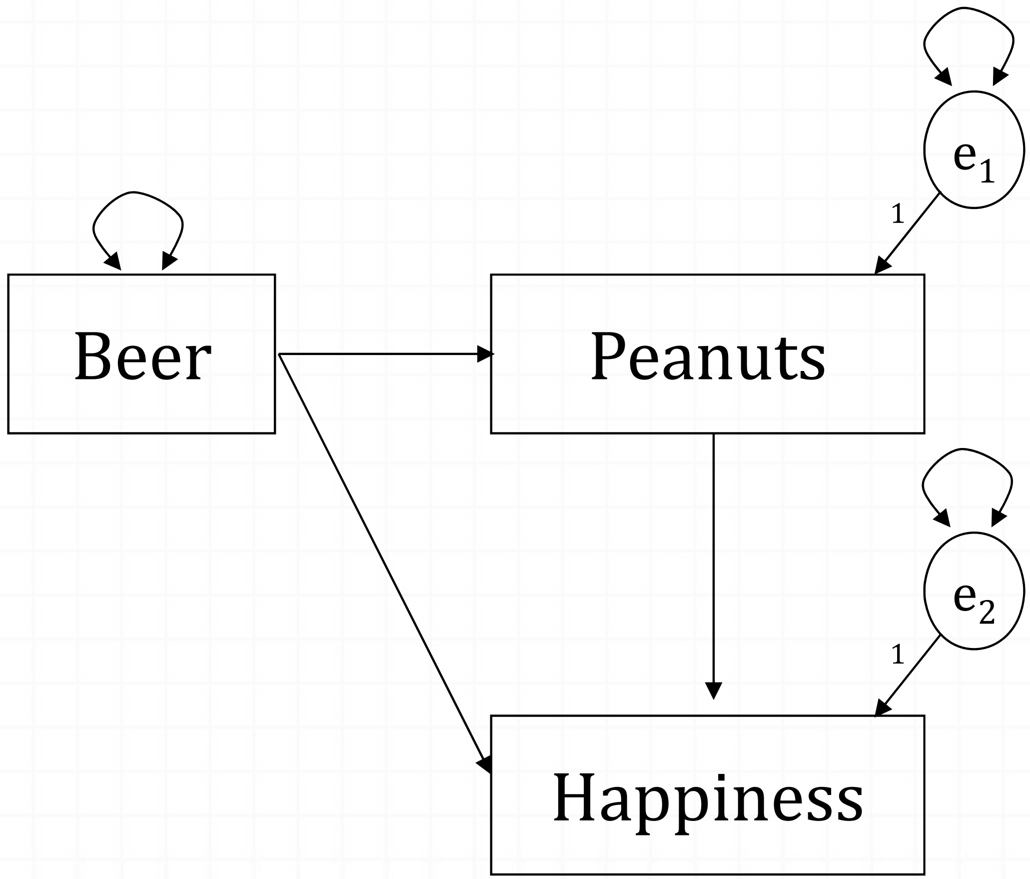


Multiple outcomes / correlated errors

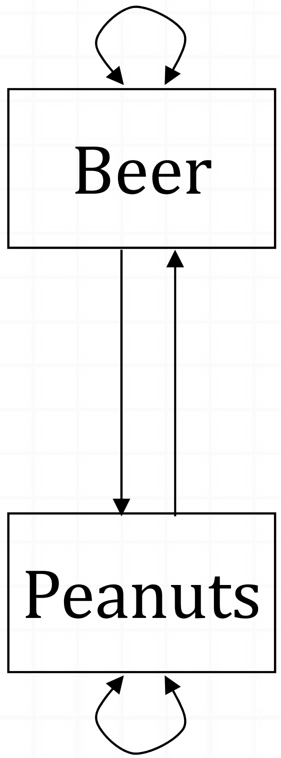


Unmeasured exposures explain Part of the residual association between happiness and windy-pops

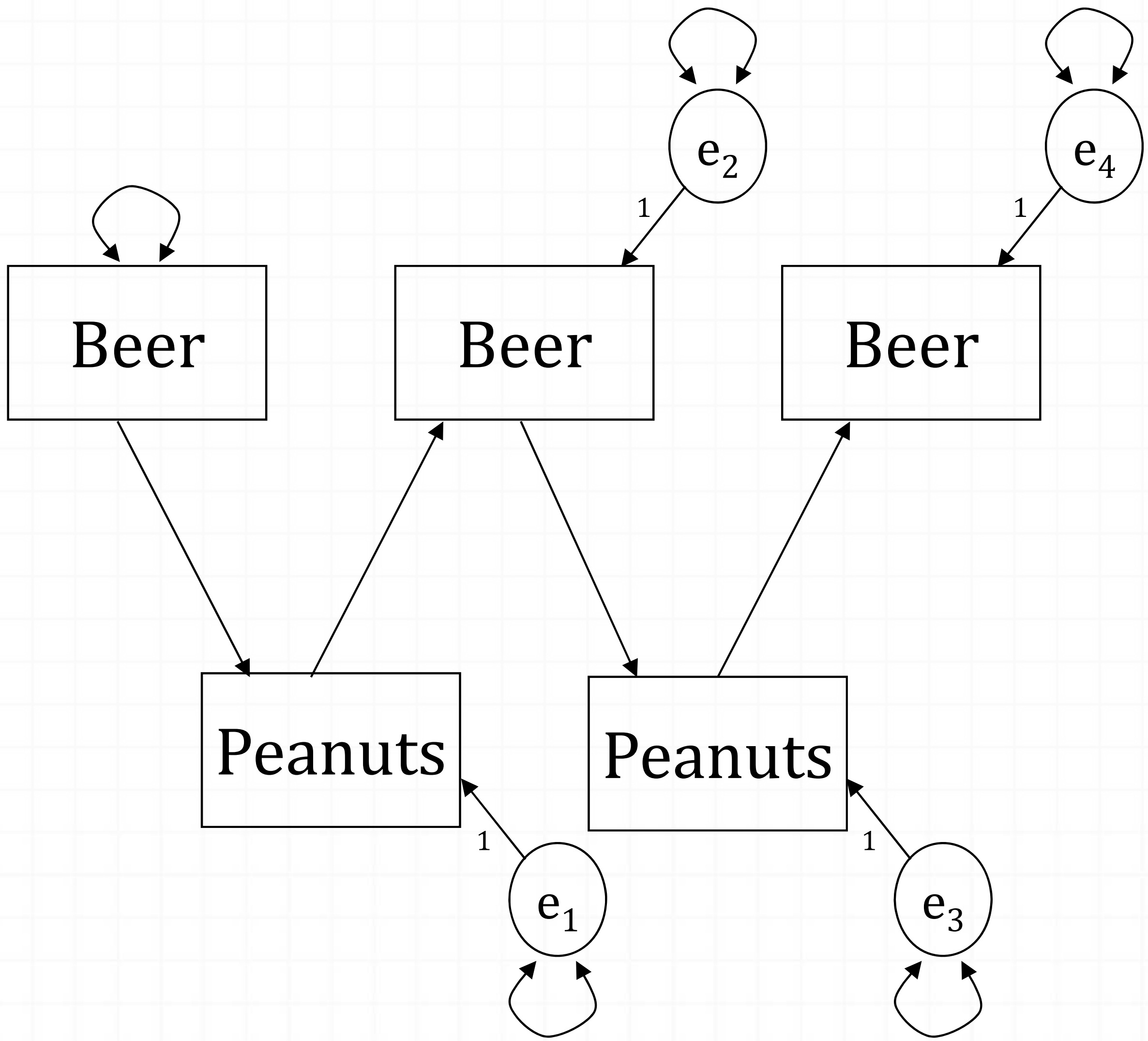
Indirect Effects



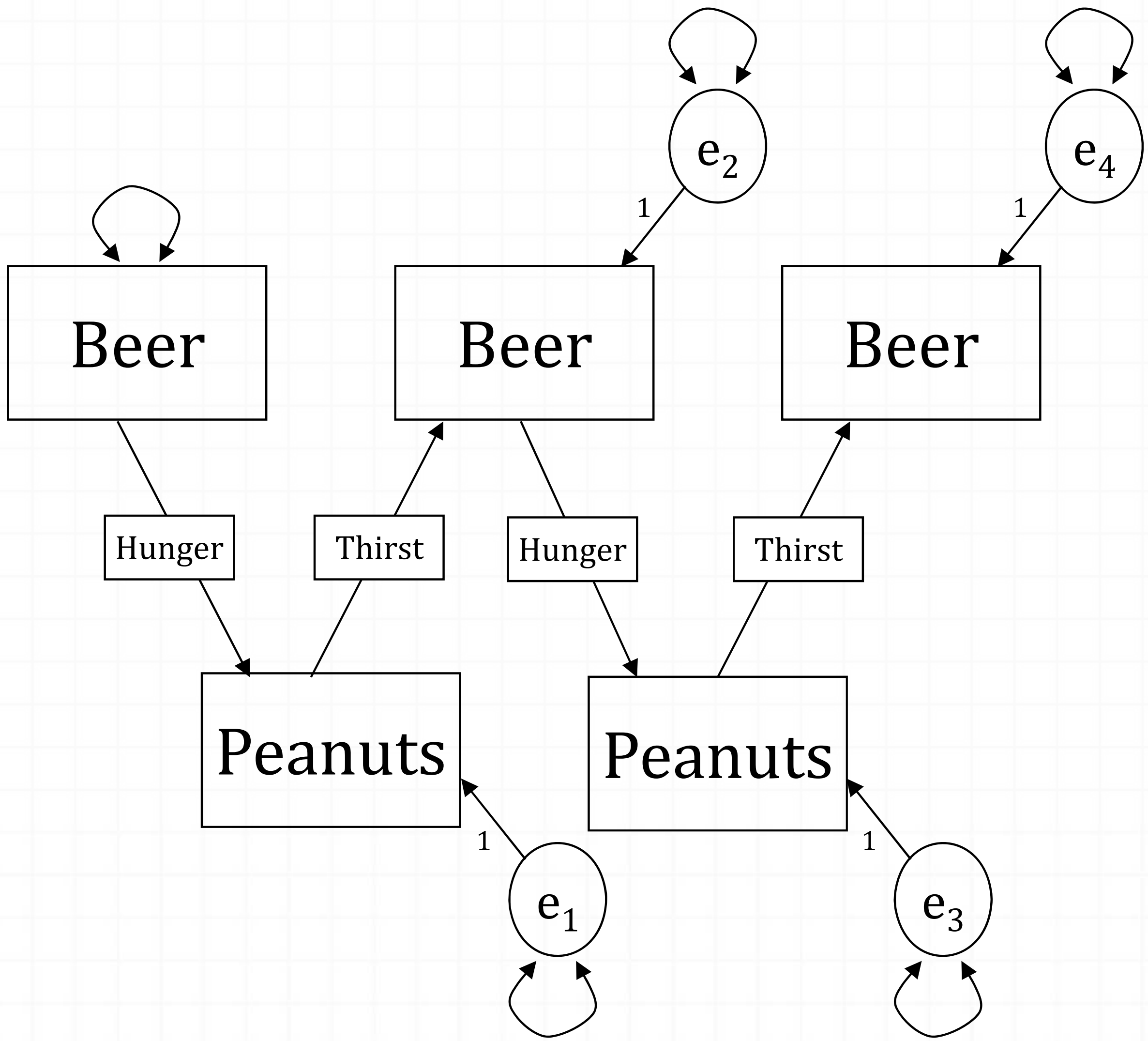
Bi-directional effects



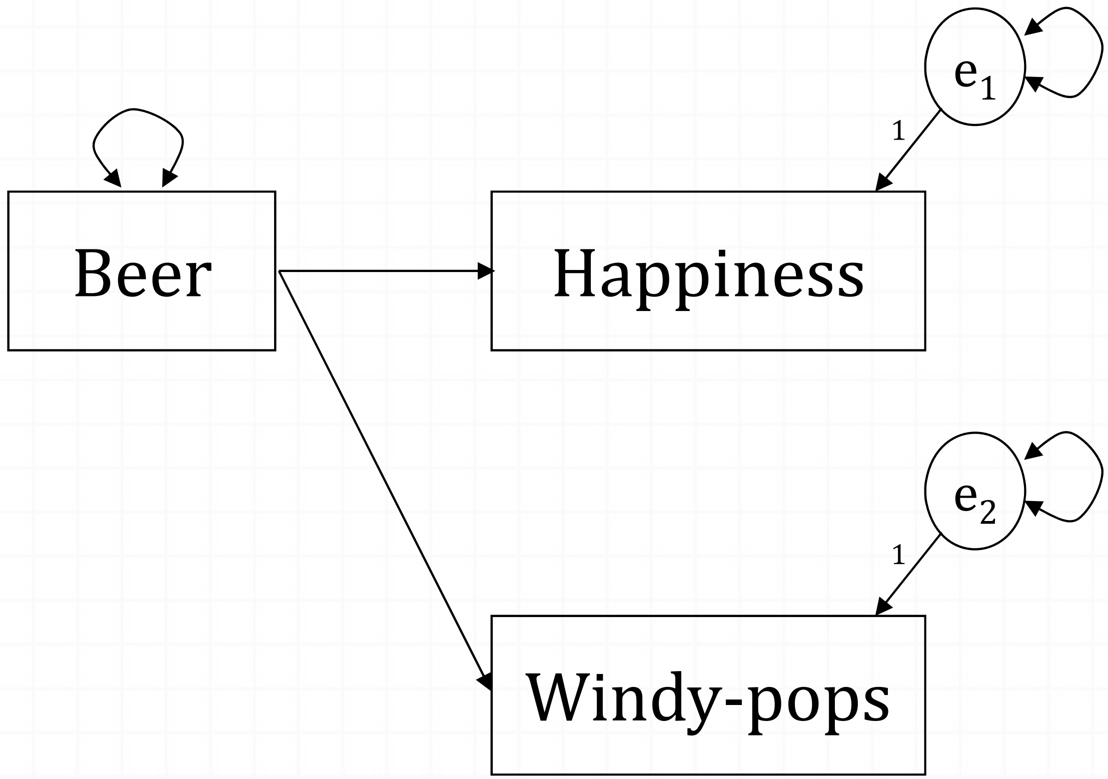
Bi-directional effects – the reality?



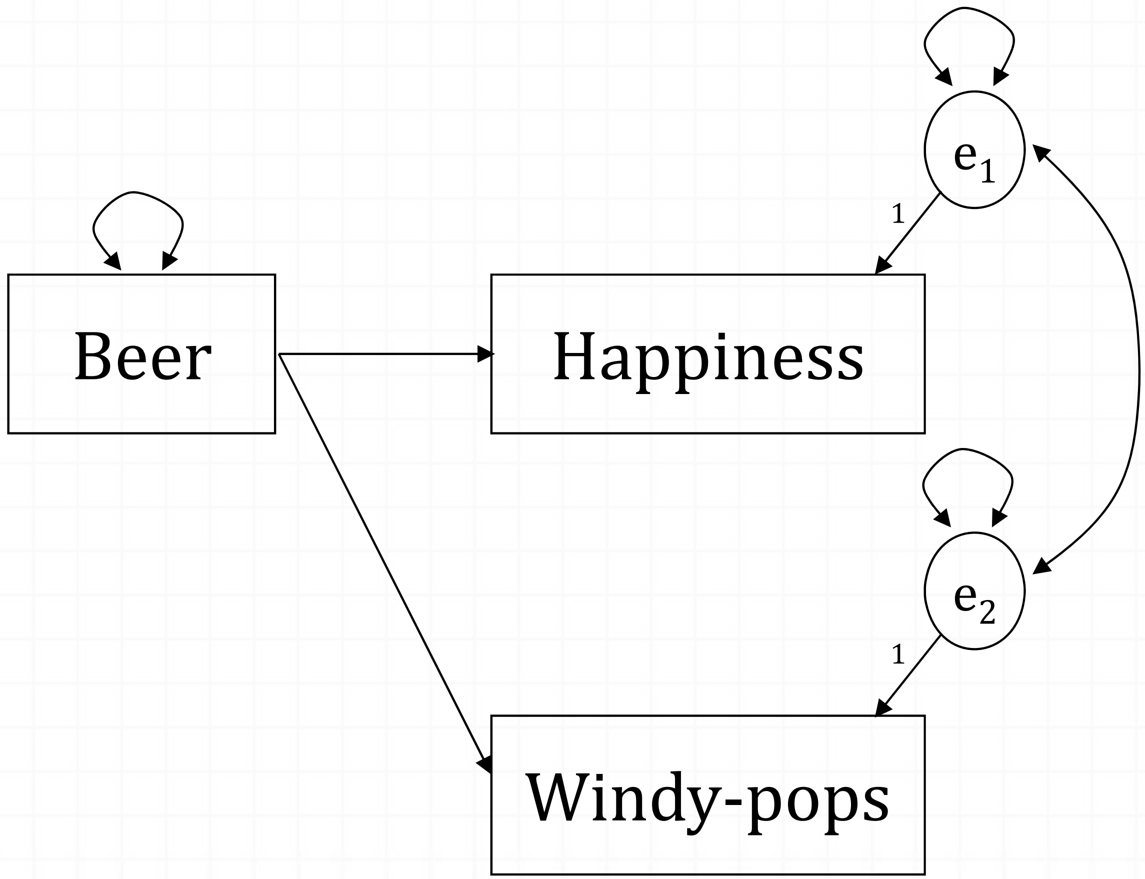
Bi-directional effects – the reality?



Recursive models



Also considered recursive



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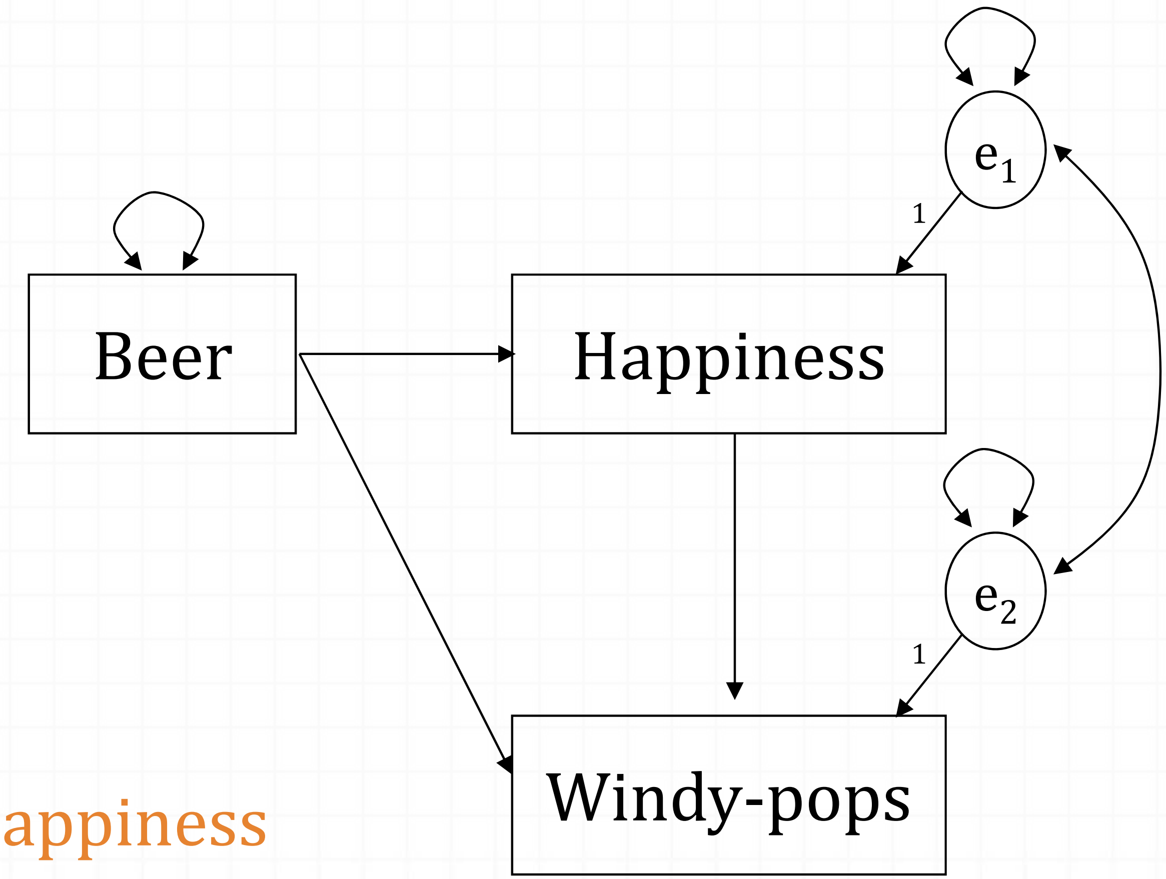
Considered non-recursive



Pathway from happiness to windy-pops and back again to happiness

Identification

- The aim of a model is to simplify the data
- The information we put IN should ideally be more than the parameters we get OUT
- Otherwise we've just re-packaged what we started with

- At best we have a model that teaches us little
- At worst we don't even get that

A simple example

- The equation

$$X_1 + X_2 = 5$$

has more unknowns (X_1, X_2) than information (5)

- There are an infinite number of solutions (values of X_1, X_2) that would satisfy this

A simple example

- What if we add another equation?

$$X_1 + X_2 = 5$$
$$2X_1 + 2X_2 = 10$$

- There is still no unique solution as equations are linearly dependent

A simple example

$$\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \end{pmatrix}$$

A*X = B

$(A)^{-1}A*X = (A)^{-1}B$

$X = (A)^{-1}B$

Cannot solve for X
A is non-invertible or
non-positive definite

A simple example

- What if they weren't linearly dependent??

$$X_1 + X_2 = 5$$
$$2X_1 + X_2 = 8$$

- There is now a unique solution: $X_1 = 3$, $X_2 = 2$

- This model is just-identified
- Information in and parameters out is balanced
- Given the equations & the X_i the 5,8 are reproducible

A simple example

- Three equations:-

$$X_1 + X_2 = 5$$
$$2X_1 + X_2 = 8$$
$$3X_1 + X_2 = 12$$

- There is now more information than unknown parameters

- This model is over-identified

A simple example

	Observed	$X_1=2, X_2=3$	$X_1=3, X_2=3$	$X_1=2.5, X_2=3$	$X_1=2.75, X_2=3$
$X_1 + X_2$	5	5	6	5.5	5.75
$2X_1 + X_2$	8	7	9	8	8.5
$3X_1 + X_2$	12	9	12	10.5	11.25
Sum of squared differences	-	0+1+9=10	1+1+0=2	2.5	1.375

A simple example

- Iterate towards a solution that minimises chosen statistic – the sums of squared differences between observed and predicted values

- Over-identified => one degree of freedom to test adequate of simplified model (assuming distribution of sum of squares is known)

What about in path analysis/SEM?

- The data is the covariance matrix
- And sometimes the means as well

- Covariance matrix for 5 variables contains (5*6)/2=15 elements
- Sample size does not affect this number!

Identification in SEM

- If every model parameter can be expressed as a unique function of the terms of the population covariance matrix such that the statistical criterion to be minimised (e.g. the sum of squared differences) is also satisfied.

- Recursive models – always identified
- Non-recursive models – more complicated

Empirical Identification

- Model identification can be assessed prior to data collection
- The data can bring a nasty surprise!

- Two measures strongly collinear
- Data very weakly correlated (~ zero cells in cov matrix)
- Out of bounds elements (pairwise deletion)
- Empirically under-identified

Time for an example

OF
GE

Pathways Between Internalized Stigma and Outcomes Related to Recovery in Schizophrenia Spectrum Disorders

Philip T. Yanos, Ph.D.
David Roe, Ph.D.
Keith Markus, Ph.D.
Paul H. Lysaker, Ph.D.

Objective: **The mechanisms by which internalized stigma affects outcomes related to recovery among people with severe mental illness have yet to be explicitly studied. This study empirically evaluated a model for how internalized stigma affects important outcomes related to recovery. *Methods:* A total of 102 persons with schizophrenia spectrum disorders completed measures of internalized stigma, awareness of mental illness, psychiatric symptoms, self-esteem, hopefulness, and coping. Path analyses tested a predicted model and an alternative model for the relationships between the variables. *Results:* Results from model 1 supported the view that internalized stigma increases avoidant coping, active social avoidance, and depressive symptoms and that these relationships are mediated by the impact of internalized stigma on hope and self-esteem. Results from model 2 replicated significant relationships from model 1 but also supported the hypothesis that positive symptoms may influence hope and self-esteem. *Conclusions:* Findings from two models supported the hypothesis that internalized stigma affects hope and self-esteem, leading to negative outcomes related to recovery. It is recommended that interventions be developed and tested to address the important effects of internalized stigma on recovery. (*Psychiatric Services* 59:1437–1442, 2008)**

proving both subjective and objective outcomes in this population (5,6).

A type of identity transformation that may affect many people with severe mental illness is the internalization of stereotypic or stigmatizing views (7–11). The state in which a person with severe mental illness loses previously held or hoped for identities (self as student, self as worker, self as parent, and so on) and adopts stigmatizing views (self as dangerous, self as incompetent, and so on) is typically referred to as "internalized stigma." As an illustration, a person with a college degree and prior aspirations to become a math teacher might conclude that he or she could never achieve this goal if he or she believes that the diagnosis of mental illness

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Population

- A total of 102 persons (87 men and 15 women) had diagnoses of schizophrenia spectrum disorders (68 with schizophrenia and 34 with schizoaffective disorder), confirmed with the Structured Clinical Interview for DSM-IV.

- They were recruited from a comprehensive day hospital at a Veterans Affairs medical center (N=70) and local community mental health center (N=32) for a study of the effects of cognitive-behavioral therapy on vocational rehabilitation.

Measures

- SUMD awareness
  - Scale for Assessing Unawareness of Mental Disorder
- Internal stigma
  - Internalized Stigma of Mental Illness Scale
- Hope and self-esteem
  - Beck Hopelessness Scale / Rosenberg Self-Esteem Scale
- Avoidant coping
  - Ways of Coping Questionnaire
- PANNS social avoidance (single item)
- PANNS depression (single item)
- PANNS positive symptoms
  - a factor-analytically derived component (positive symptoms, such as hallucinations and delusions)

The data

Table 1

Correlations, variances, and covariances among variables included in path analysis of 102 patients with severe mental illness

Measure[a]	1	2	3	4	5	6	7
1. SUMD awareness	7.32[b]	–.51	.76	–.04	–.55	–1.11	–.17
2. Internal stigma	–.18	1.17[b]	–1.16	.13	.39	.31	1.12
3. Hope and self-esteem	.16	–.59*	3.23[b]	–.44	–1.10	–1.21	–.17
4. Avoidant coping	–.03	.24*	–.50*	.24[b]	.14	.18	–.04
5. PANNS social avoidance	–.16	.28*	–.49*	.23*	1.58[b]	.84	1.95
6. PANNS depression	–.25*	.17	–.41*	.22*	.40*	2.74[b]	1.42
7. PANNS positive symptoms	–.01	.24*	–.32*	–.09	.35*	.19	19.57[b]

[a] SUMD, Scale for Assessing Unawareness of Mental Disorder; PANNS, Positive and Negative Syndrome Scale.

[b] Variances are noted on the diagonal. Correlations are shown below the main diagonal, and covariances are shown above the diagonal.

*p<.05

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The data – warning!!

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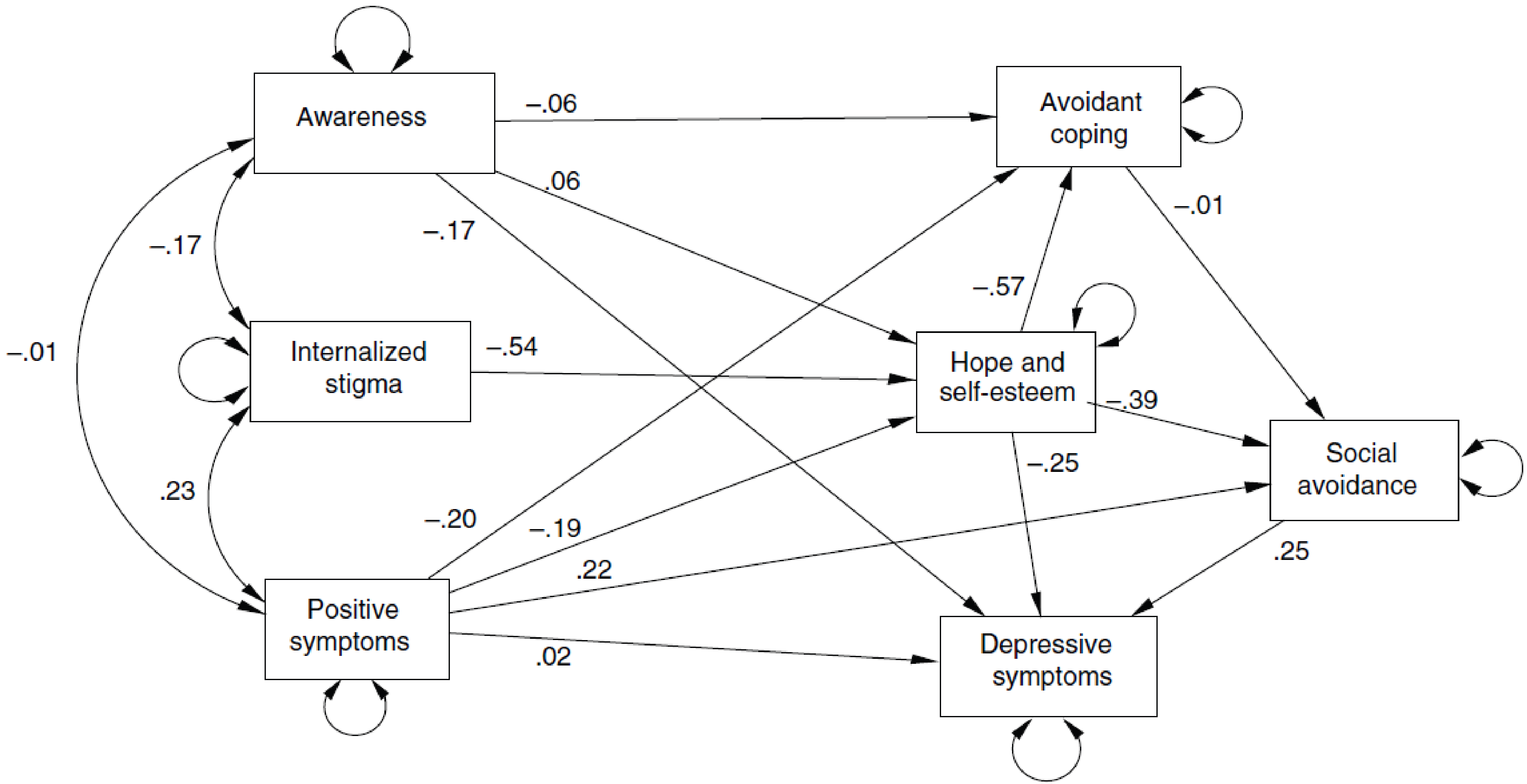
*p<.05

A proposed model (model 2 in paper)

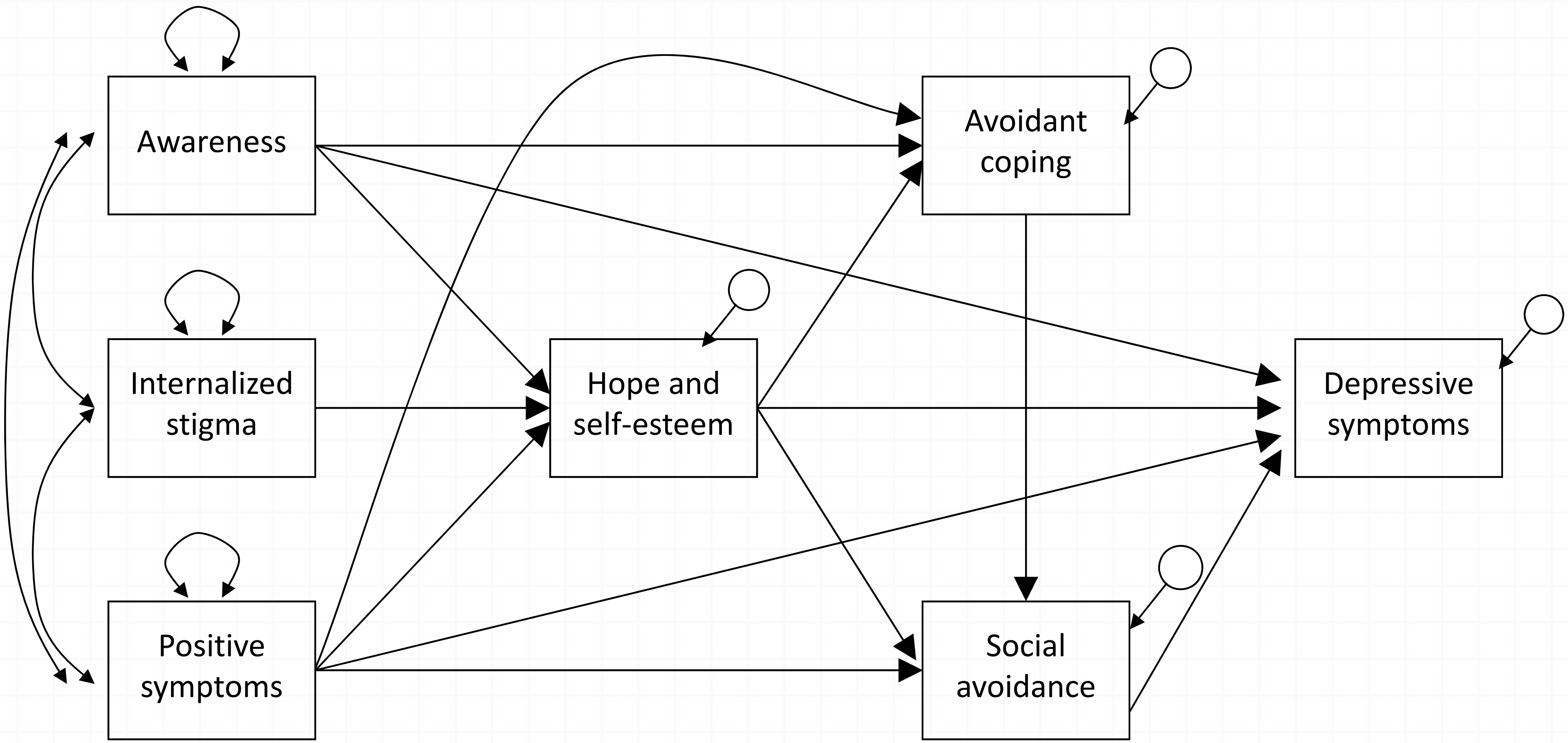
Figure 3

Path model 2, where positive symptoms of schizophrenia are treated as input[a]



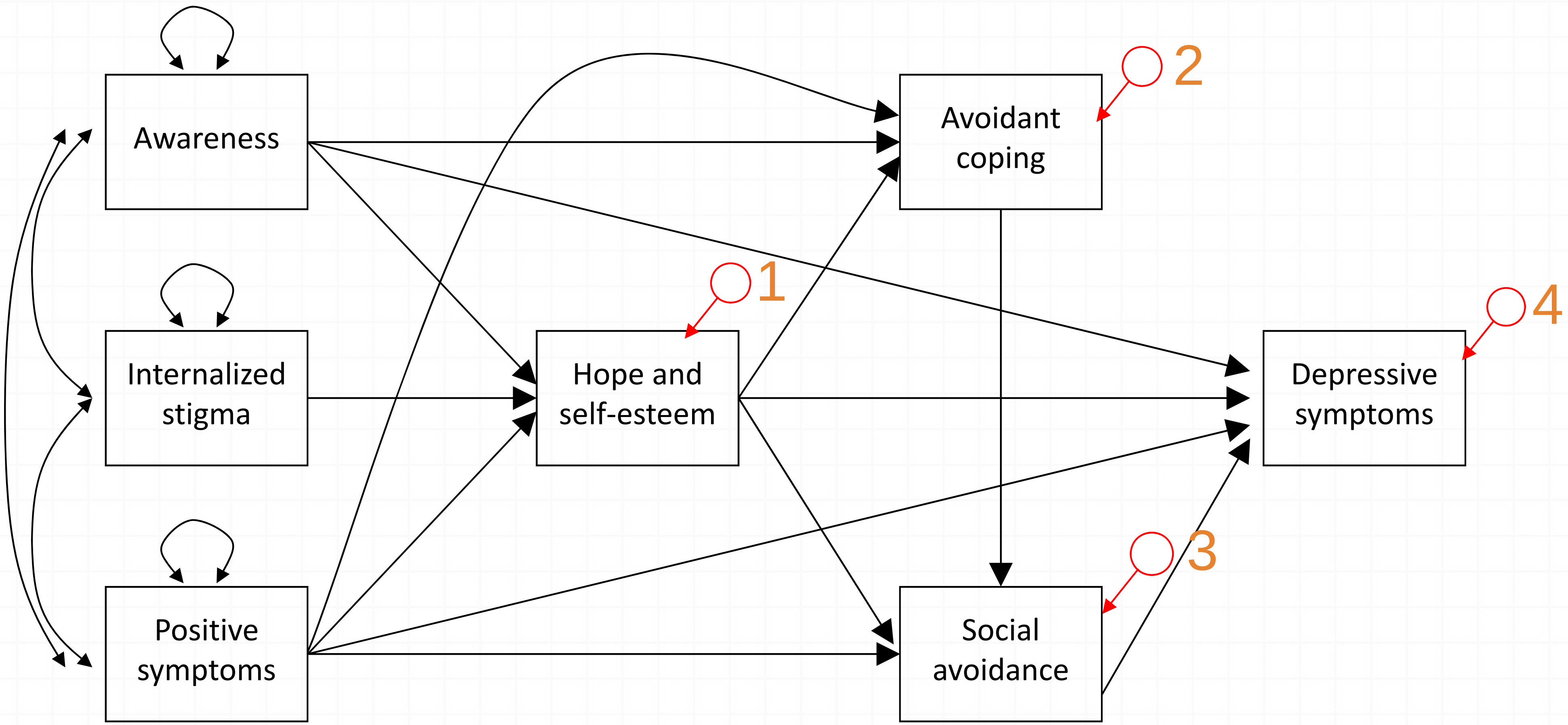
[a] N=102. Standardized coefficients are presented.

A *tweaked* model diagram

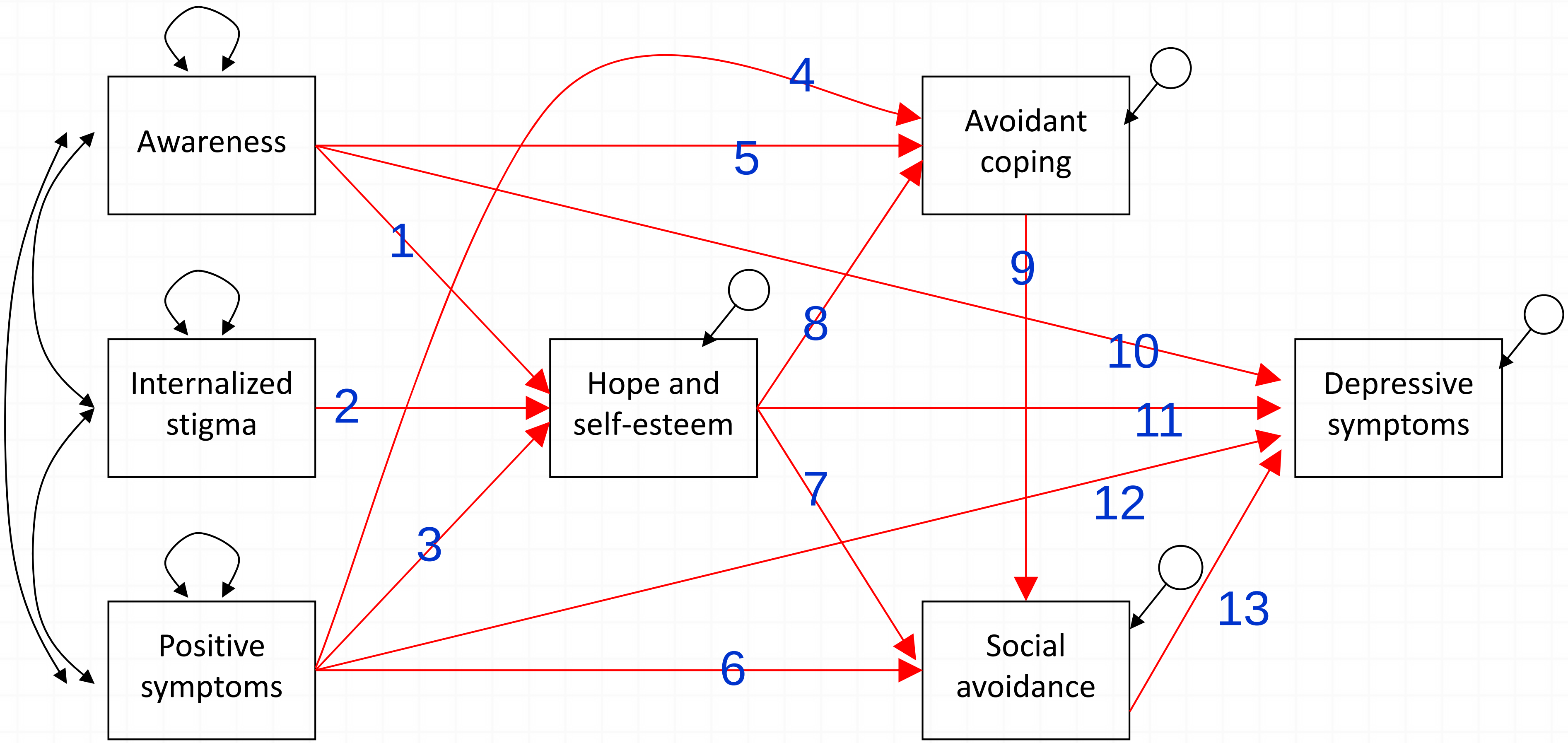


No effects flowing upstream
Residuals included for dependent variables

FOUR estimated residual variances



THIRTEEN estimated associations



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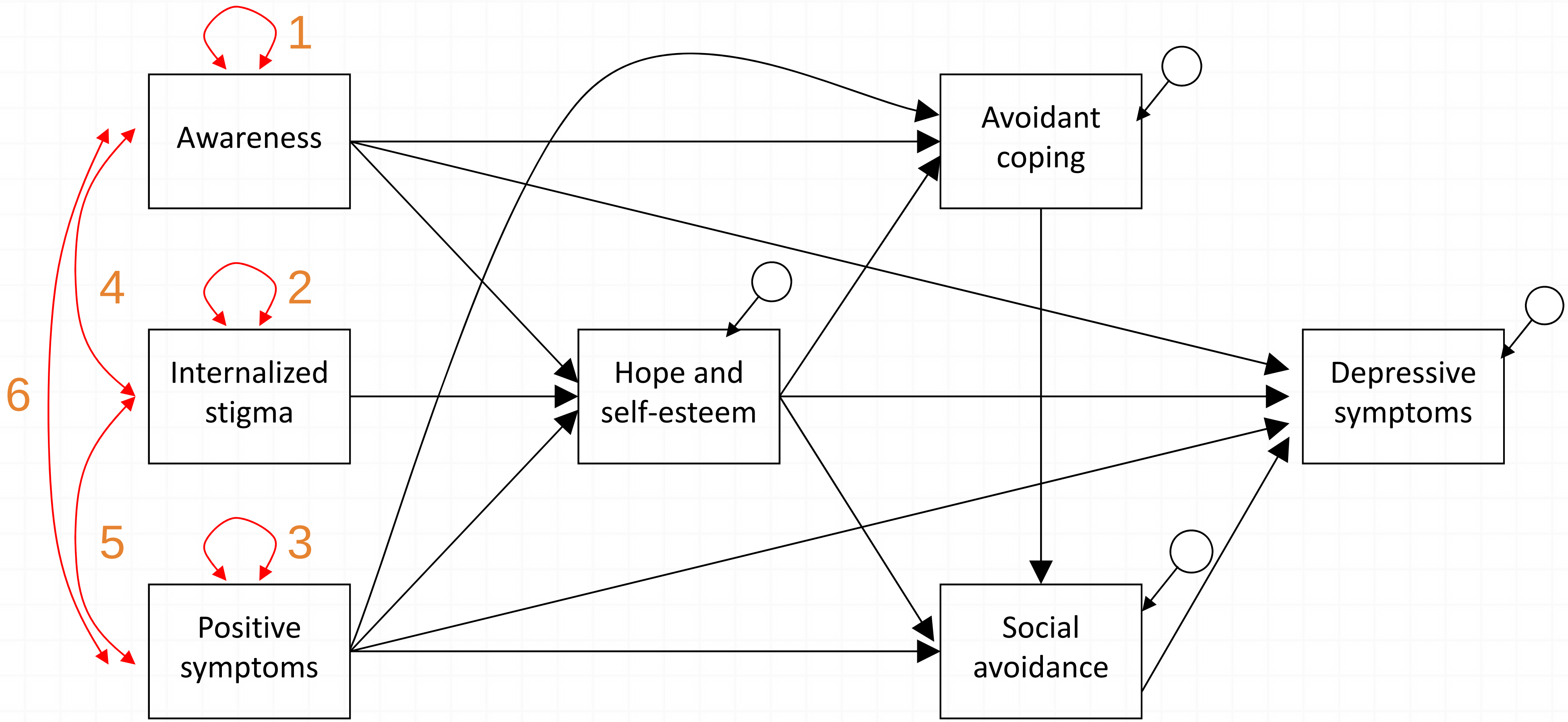
SIX estimated exogenous (co)variances



Identified?

7.32[b]	–.51	.76	–.04	–.55	–1.11	–.17
–.18	1.17[b]	–1.16	.13	.39	.31	1.12
.16	–.59*	3.23[b]	–.44	–1.10	–1.21	–.17
–.03	.24*	–.50*	.24[b]	.14	.18	–.04
–.16	.28*	–.49*	.23*	1.58[b]	.84	1.95
–.25*	.17	–.41*	.22*	.40*	2.74[b]	1.42
–.01	.24*	–.32*	–.09	.35*	.19	19.57[b]

Covariance matrix has 7+6+5+4+3+2+1 = (7*8)/2 = **28** unique items

Proposed model has 13+4+6 = **23** parameters

Model is **over-identified** (provided it is recursive)

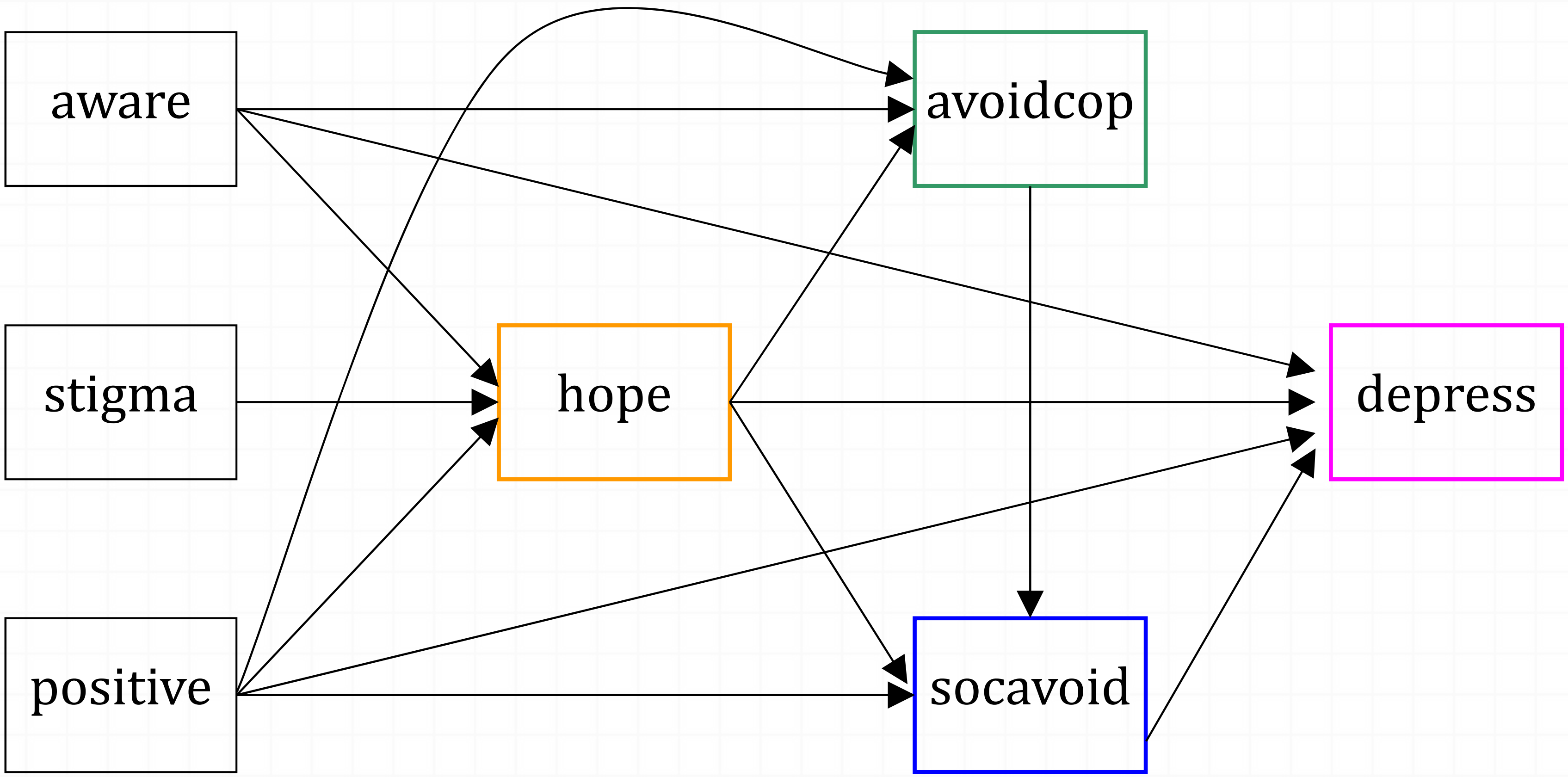
5 degrees of freedom left over to test model

Just to add a little confusion….

- When fitting this model in Mplus, only **17** parameters would be presented and not **23**
- Exogenous covariance matrix not part of default output

- The same occurs when fitting a regression model – we are not usually interested in the associations within our covariates
- This doesn't mean they are constrained to be zero

- These values **can** be requested + the model will not be affected, neither will the d.f. for model testing (in this case **5**)

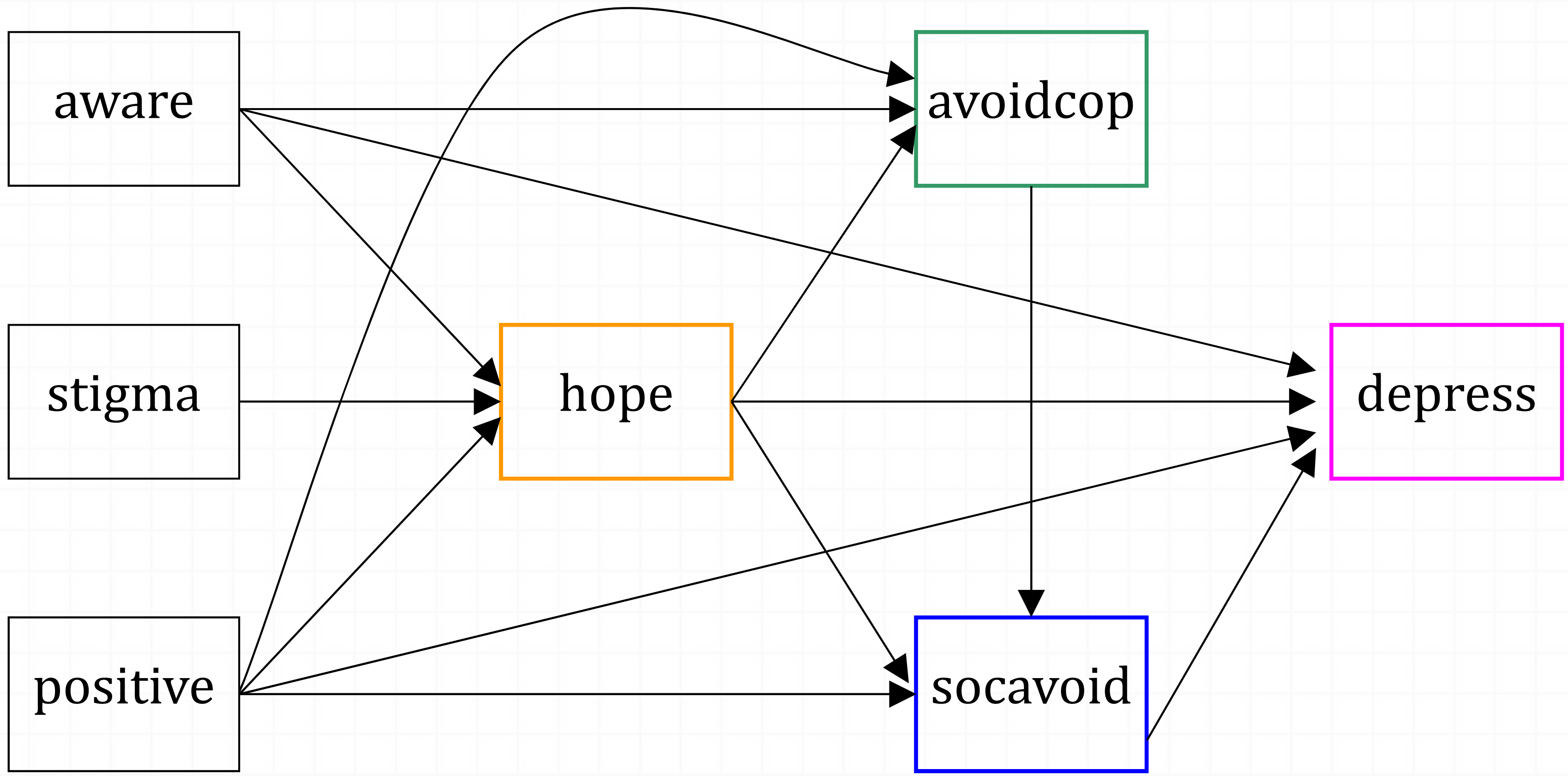


socavoid on avoidcop; **socavoid** on hope; **socavoid** on positive;

avoidcop on aware; **avoidcop** on positive; **avoidcop** on hope;

hope on aware; **hope** on stigma; **hope** on positive;

depress on socavoid; **depress** on hope; **depress** on aware; **depress** on positive;



socavoid on avoidcop hope positive;

avoidcop on aware positive hope;

hope on aware stigma positive;

depress on socavoid hope aware positive;

Full syntax

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```
DATA:
  FILE = "sz input matrix2.txt";
  TYPE = STD CORRELATION;
  NGROUPS = 1;
  NOBSERVATIONS = 102;

VARIABLE:
  NAMES = aware stigma hope avoidcop socavoid depress positive;
  USEVARIABLES = aware stigma hope avoidcop socavoid depress positive;

MODEL:
   socavoid on avoidcop hope positive;
   avoidcop on aware positive hope;
   hope on aware stigma positive;
   depress on socavoid hope aware positive;

   ! residual variances for endogenous variables - unnecessary
   hope avoidcop socavoid depress;

   ! exogenous covariance matrix - unnecessary
   aware stigma positive;
   aware with stigma positive;
   stigma with positive;

OUTPUT:
   standardized residual modindices(3.8);;
```

Model fit

```
TESTS OF MODEL FIT

Chi-Square Test of Model Fit
          Value                                3.475
          Degrees of Freedom                       5
          P-Value                             0.6271

Chi-Square Test of Model Fit for the Baseline Model
          Value                              156.188
          Degrees of Freedom                      18
          P-Value                             0.0000

CFI/TLI

          CFI                                  1.000
          TLI                                  1.040
```

Model fit

```
Loglikelihood
          H0 Value                        -1251.477
          H1 Value                        -1249.739

Information Criteria
          Number of Free Parameters              23
          Akaike (AIC)                     2548.954
          Bayesian (BIC)                   2609.329
          Sample-Size Adjusted BIC         2536.680

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                            0.000
          90 Percent C.I.                     0.000  0.114
          Probability RMSEA <= .05            0.742

SRMR (Standardized Root Mean Square Residual)
          Value                               0.027
```

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Covariances/Correlations/Residual Correlations

	HOPE	AVOIDCOP	SOCAVOID	DEPRESS	AWARE	STIGMA	POSITIVE
HOPE	3.229						
AVOIDCOP	-0.440	0.240					
SOCAVOID	-1.107	0.142	1.580				
DEPRESS	-1.219	0.178	0.832	2.739			
AWARE	0.778	-0.040	-0.544	-1.120	7.322		
STIGMA	-1.147	0.127	0.381	0.304	-0.527	1.171	
POSITIVE	-2.544	-0.195	1.946	1.391	-0.120	1.149	19.57

Model Estimated Covariances/Correlations/Residual Correlations

	HOPE	AVOIDCOP	SOCAVOID	DEPRESS	AWARE	STIGMA	POSITIVE
HOPE	3.198						
AVOIDCOP	-0.436	0.238					
SOCAVOID	-1.096	0.140	1.565				
DEPRESS	-1.207	0.150	0.791	2.691			
AWARE	0.770	-0.039	-0.222	-1.009	7.250		
STIGMA	-1.136	0.144	0.404	0.461	-0.522	1.159	
POSITIVE	-2.519	-0.193	1.927	1.378	-0.119	1.138	19.38

Standardized mean residual

The standardized residuals are computed as follows. The standardized mean residual is

$$\frac{m_i - \hat{\mu}_i}{\sqrt{Var(m_i - \hat{\mu}_i)}}. \tag{13}$$

By Hausman's (1978) theorem, under the assumption of correct model specification

$$Var(m_i - \hat{\mu}_i) = Var(m_i) - Var(\hat{\mu}_i) \tag{14}$$

Hausman, J. (1978), Specification tests in econometrics., Econometrica 46(6), 1251-71.

From Mplus tech appendix

Standardized covariance residual

The standardized covariance residual is

$$\frac{s_{ij} - \hat{\sigma}_{ij}}{\sqrt{Var(s_{ij} - \hat{\sigma}_{ij})}} \tag{15}$$

and again by Hausman's (1978) theorem

$$Var(s_{ij} - \hat{\sigma}_{ij}) = Var(s_{ij}) - Var(\hat{\sigma}_{ij}) \tag{16}$$

Hausman, J. (1978), Specification tests in econometrics., Econometrica 46(6), 1251-71.

From Mplus tech appendix

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Problem with standardized residuals

One problem with Hausman's (1978) approach to computing the residual variance is that sometimes the variance estimates given by (14) and (16) can be negative. In that case the standardized residual is not computed and Mplus prints 999. Typically in such situation the normalized residual can be used.

Note also that the normalized residual is always smaller by absolute value than the standardized, i.e., the normalized residual is a more conservative test. Under the null hypothesis the standardized residual should have a standard normal distribution and any deviation from that would indicate model misfit. Under the null hypothesis the normalized residuals should have distribution smaller than the standard normal distribution and any deviation from that would indicate model misfit.

From Mplus tech appendix

Standardized residuals

Standardized Residuals (z-scores) for Covariances/Correlations/Residual Corr

	HOPE	AVOIDCOP	SOCAVOID	DEPRESS	AWARE	STIGMA	POSITIVE
HOPE	999.000						
AVOIDCOP	-0.019	0.019					
SOCAVOID	-0.012	0.009	0.012				
DEPRESS	0.000	0.453	0.806	0.412			
WARE	0.010	-0.002	-1.123	-0.942	0.027		
STIGMA	-0.057	-0.551	-0.297	-1.318	-0.004	0.000	
POSITIVE	999.000	999.000	999.000	-0.001	0.011	999.000	999.000

Normalized residuals

Normalized Residuals for Covariances/Correlations/Residual Correlations

	HOPE	AVOIDCOP	SOCAVOID	DEPRESS	AWARE	STIGMA	POSITIVE
HOPE	0.000						
AVOIDCOP	0.000	0.000					
SOCAVOID	0.000	0.000	0.000				
DEPRESS	0.000	0.322	0.150	0.054			
AWARE	0.000	0.000	-0.937	-0.219	0.000		
STIGMA	0.000	-0.345	-0.193	-0.898	0.000	0.000	
POSITIVE	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Modification Indices

Minimum M.I. value for printing the modification index 1.000

	M.I.	E.P.C.	Std E.P.C.	StdYX E.P.C.
ON Statements				
HOPE ON DEPRESS	1.534	-0.215	-0.215	-0.197
AVOIDCOP ON SOCAVOID	1.302	0.567	0.567	1.453
SOCAVOID ON AWARE	1.301	-0.045	-0.045	-0.097
DEPRESS ON STIGMA	1.545	-0.204	-0.204	-0.134
WITH Statements				
SOCAVOID WITH AVOIDCOP	1.301	0.634	0.634	1.495
DEPRESS WITH HOPE	1.545	-0.453	-0.453	-0.226
AWARE WITH SOCAVOID	1.410	-0.331	-0.331	-0.116
AWARE WITH DEPRESS	1.547	-3.033	-3.033	-0.789
STIGMA WITH DEPRESS	1.545	-0.215	-0.215	-0.140
POSITIVE WITH DEPRESS	1.544	3.694	3.694	0.588

Interpret Estimates

STDYX Standardization

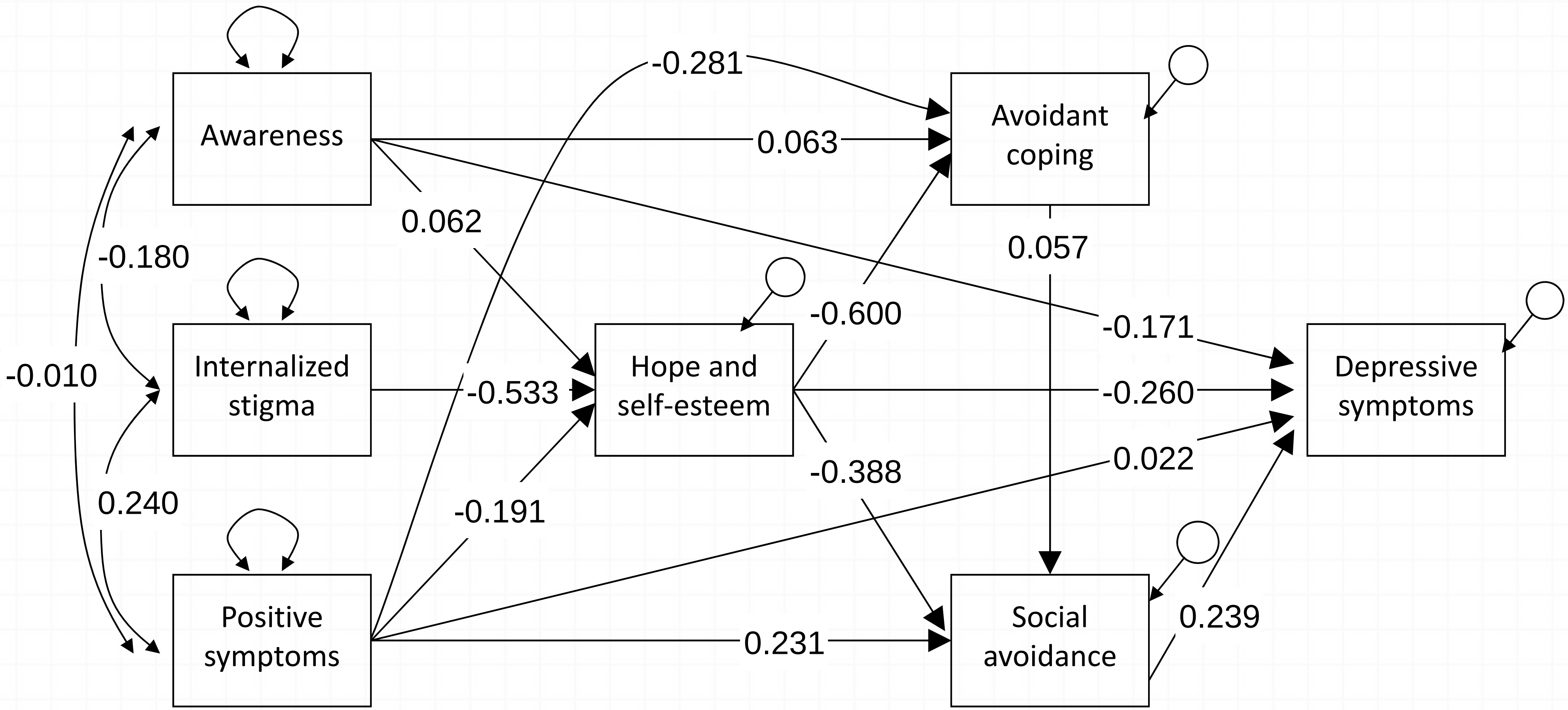
			Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
SOCAVOID	ON	AVOIDCOP	0.057	0.102	0.562	0.574
		HOPE	-0.388	0.102	-3.798	0.000
		POSITIVE	0.231	0.091	2.532	0.011
AVOIDCOP	ON	AWARE	0.063	0.082	0.766	0.444
		POSITIVE	-0.281	0.084	-3.334	0.001
		HOPE	-0.600	0.074	-8.116	0.000
HOPE	ON	AWARE	0.062	0.079	0.788	0.431
		STIGMA	-0.533	0.071	-7.530	0.000
		POSITIVE	-0.191	0.079	-2.416	0.016
DEPRESS	ON	SOCAVOID	0.239	0.100	2.394	0.017
		HOPE	-0.260	0.099	-2.614	0.009
		AWARE	-0.171	0.086	-1.976	0.048
		POSITIVE	0.022	0.094	0.237	0.813
AWARE	WITH	STIGMA	-0.180	0.096	-1.879	0.060
		POSITIVE	-0.010	0.099	-0.101	0.920
STIGMA	WITH	POSITIVE	0.240	0.093	2.572	0.010

Interpret Estimates

STDYX Standardization

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Variances				
AWARE	1.000	0.000	999.000	999.000
STIGMA	1.000	0.000	999.000	999.000
POSITIVE	1.000	0.000	999.000	999.000
Residual Variances				
HOPE	0.614	0.076	8.132	0.000
AVOIDCOP	0.676	0.076	8.878	0.000
SOCAVOID	0.716	0.076	9.477	0.000
DEPRESS	0.758	0.074	10.281	0.000

Estimated associations



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Consider near equivalent models

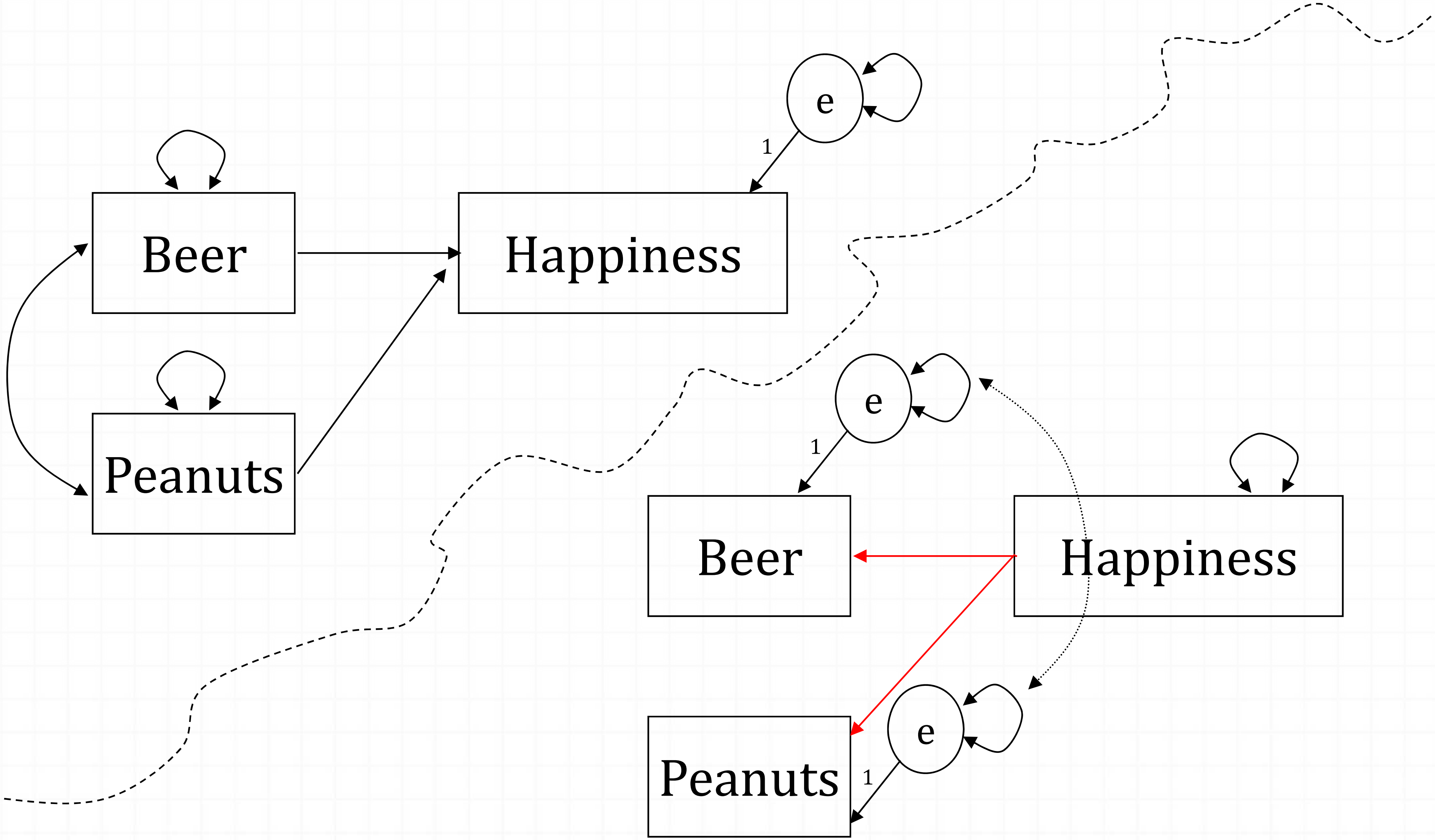


Figure 2

Path model 1, where positive symptoms of schizophrenia are treated as an outcome[a]

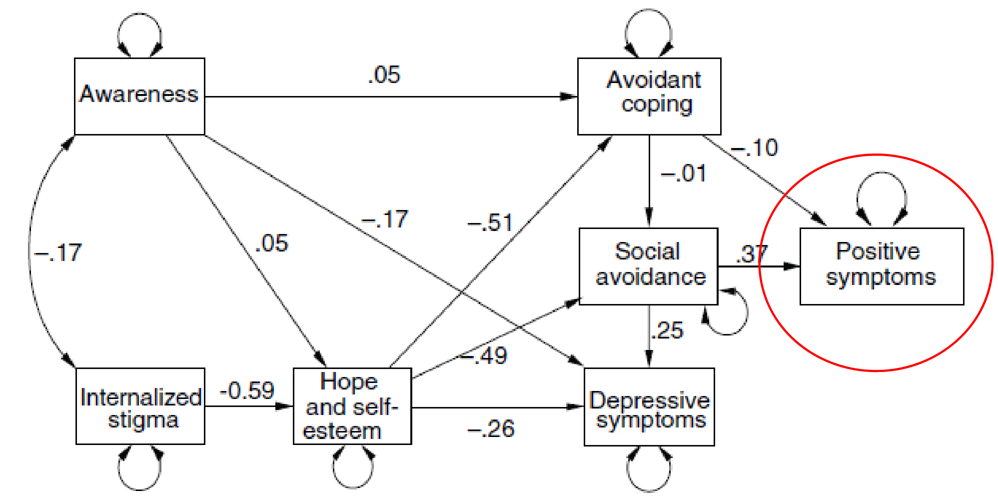
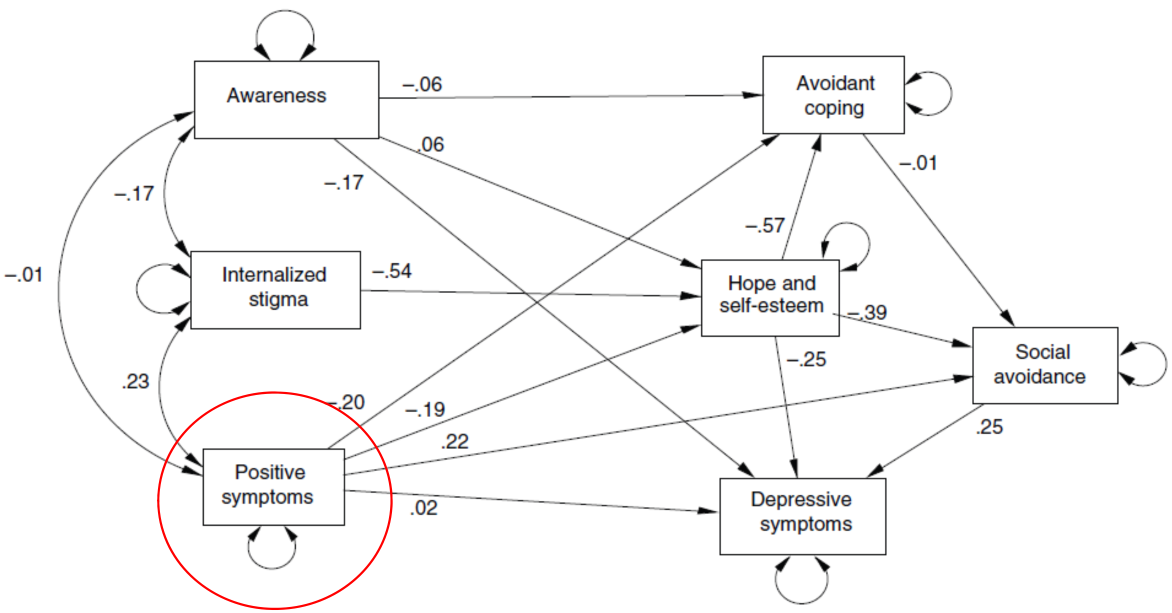


Figure 3

Path model 2, where positive symptoms of schizophrenia are treated as input[a]



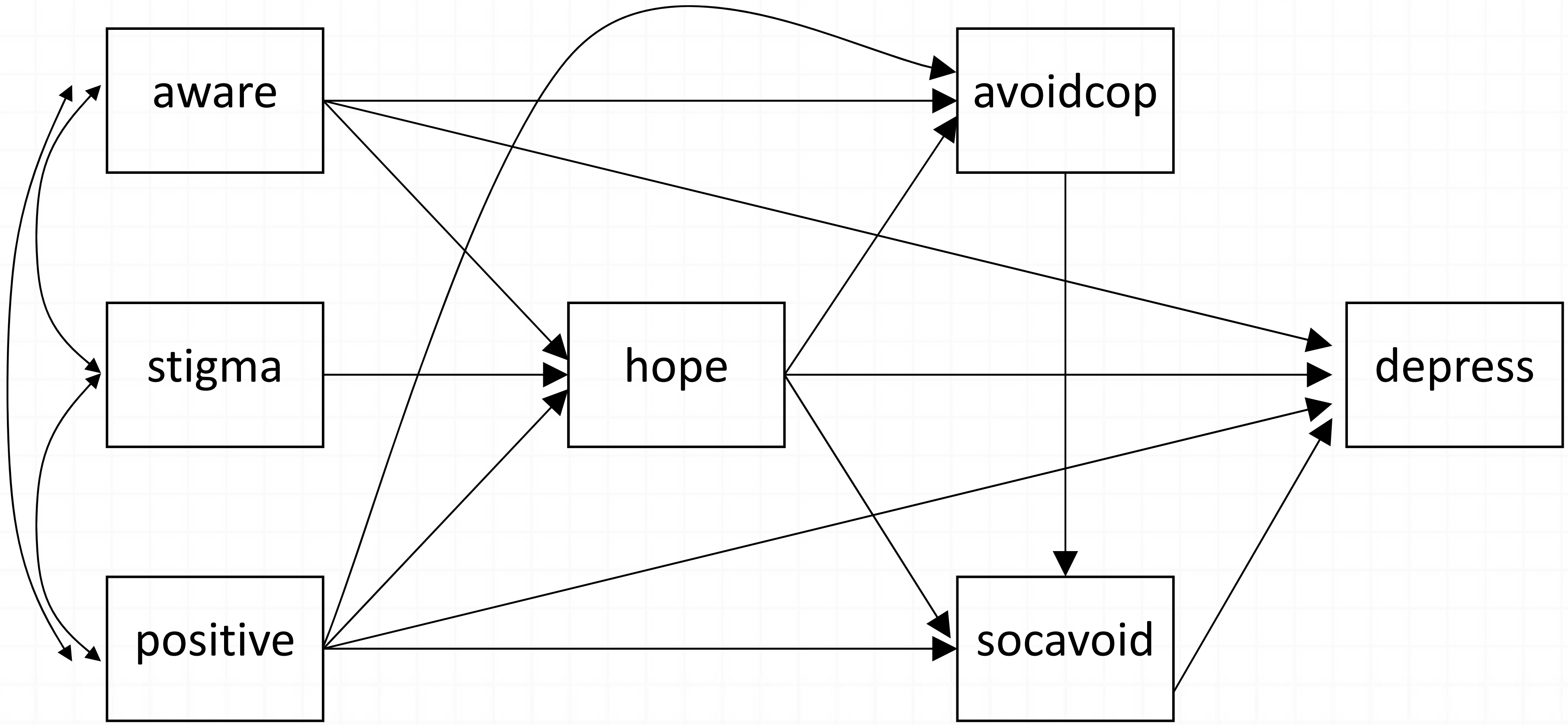
"The main difference between the two models is that the first model treats positive symptoms as an outcome whereas the second treats it as an input, or predictor of outcome."

"Model fit indices suggest that the alternative model also fit the data well."

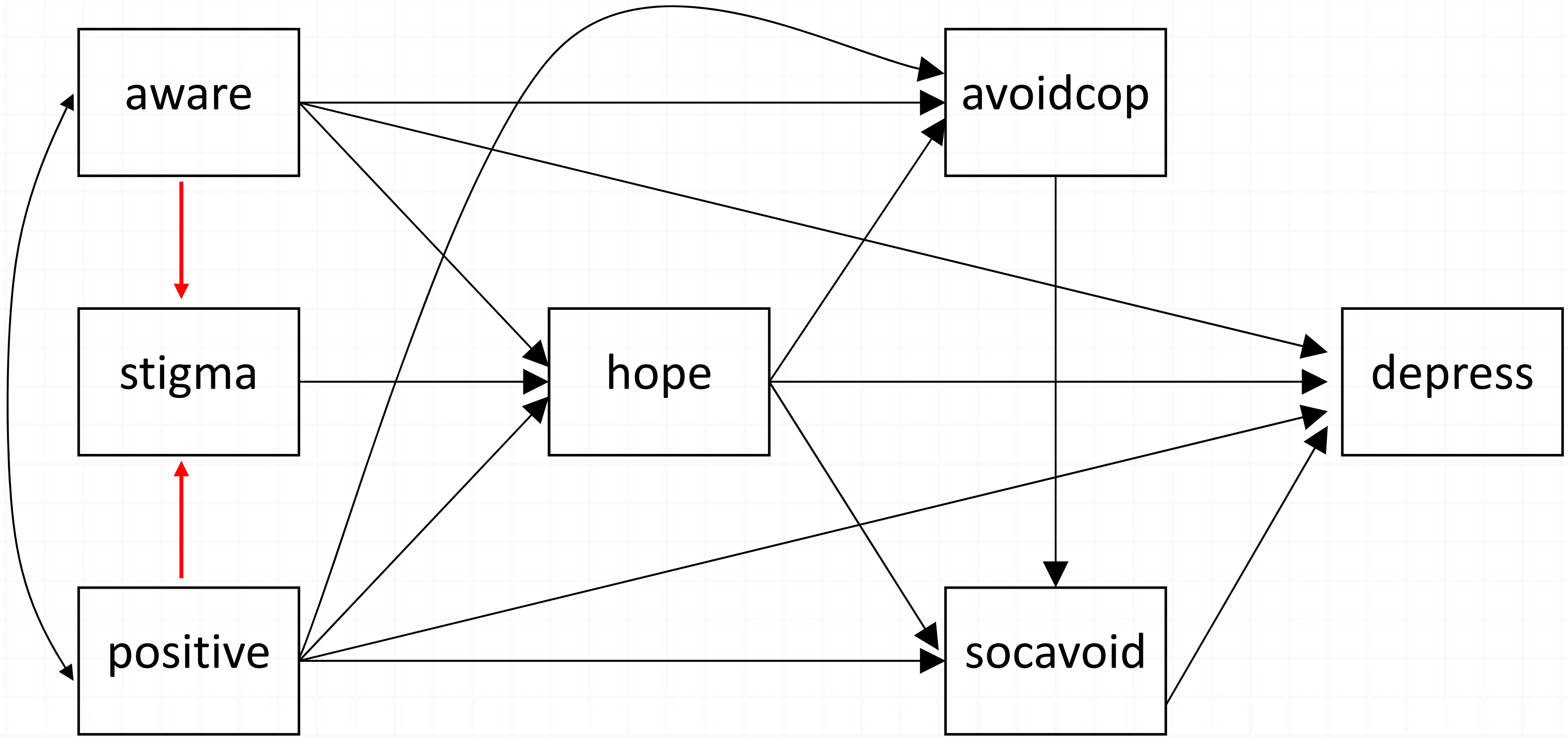
Lee-Hershberger replacing rule 1

Within a block of variables at the beginning of a model that is just-identified and with unidirectional relations to subsequent variables, direct effects, correlated disturbances, and equality-constrained reciprocal effects are interchangeable

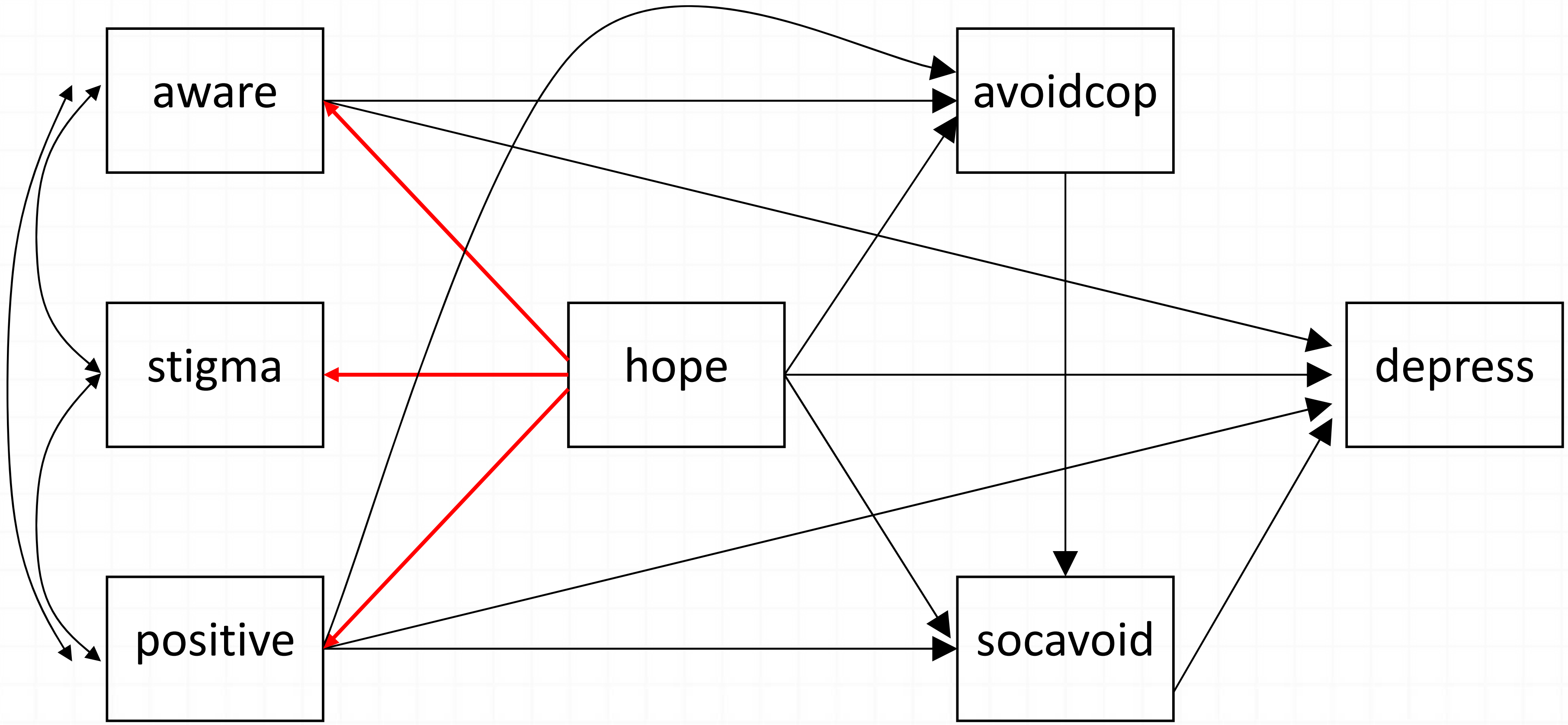
Lee-Hershberger replacing rule 1



Minor tweaks?



Or major revisions contrary to theory?



Lee-Hershberger replacing rule 2

At subsequent places in the model where two endogenous variables have the same causes and their relations are unidirectional, all of the following may be substituted for one another: $Y1 \rightarrow Y2$, $Y2 \rightarrow Y1$, $D1\ D2$, and the equality-constrained reciprocal effect $Y1\ \ Y2$

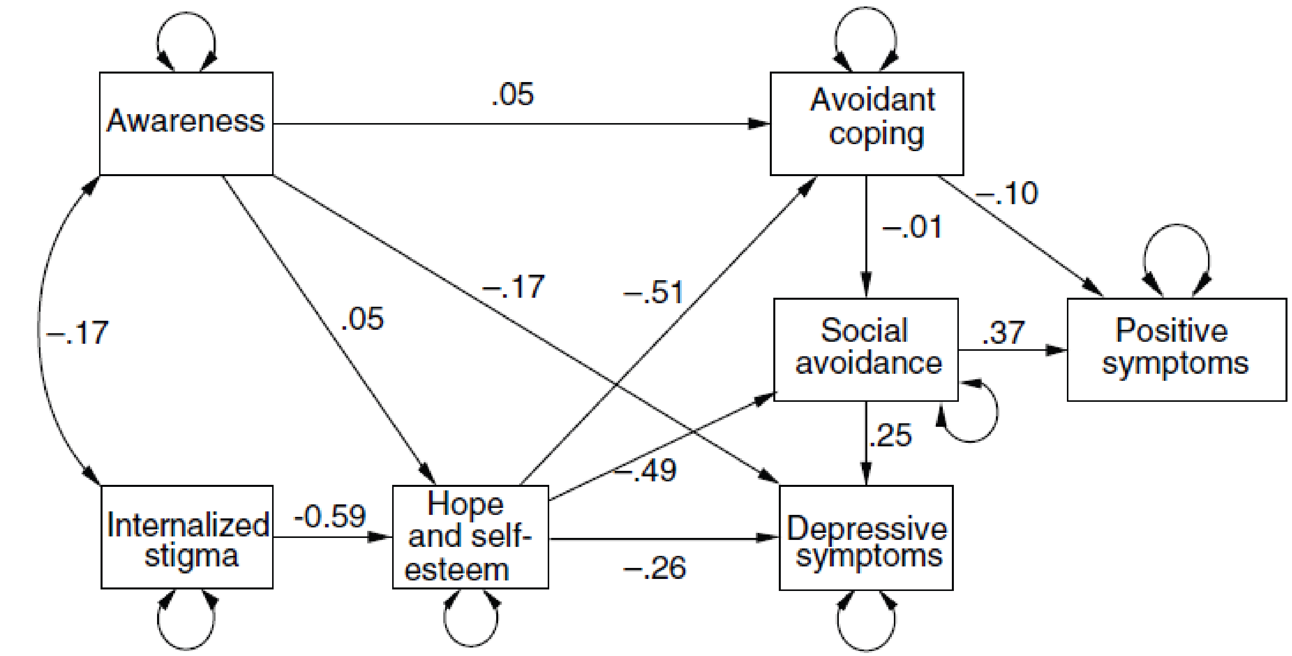
Equivalent Models

- Models with an entirely different interpretation may fit the data equally well.
- A good model fit does not give you evidence that yours was the model that generated the data
- Should always consider alternative models
- There may be many equivalent models, particularly if your model is complex
- There may be many many more near-equivalent models

Practical time

Figure 2

Path model 1, where positive symptoms of schizophrenia are treated as an outcome[a]



[a] N=102. Standardized coefficients are presented.

Convert model 1 from the schizophrenia paper into Mplus model syntax

How many parameters do you expect and of what type?

Interpret the output (that we're providing)

Path Analysis 2

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This Session

- Path Analysis Models [2]
  - Model refinement (path testing)
  - Direct and Indirect effects (mediation)
  - Mediation with binary measures
- Examples 4 – Path Analysis ~EAS temperament

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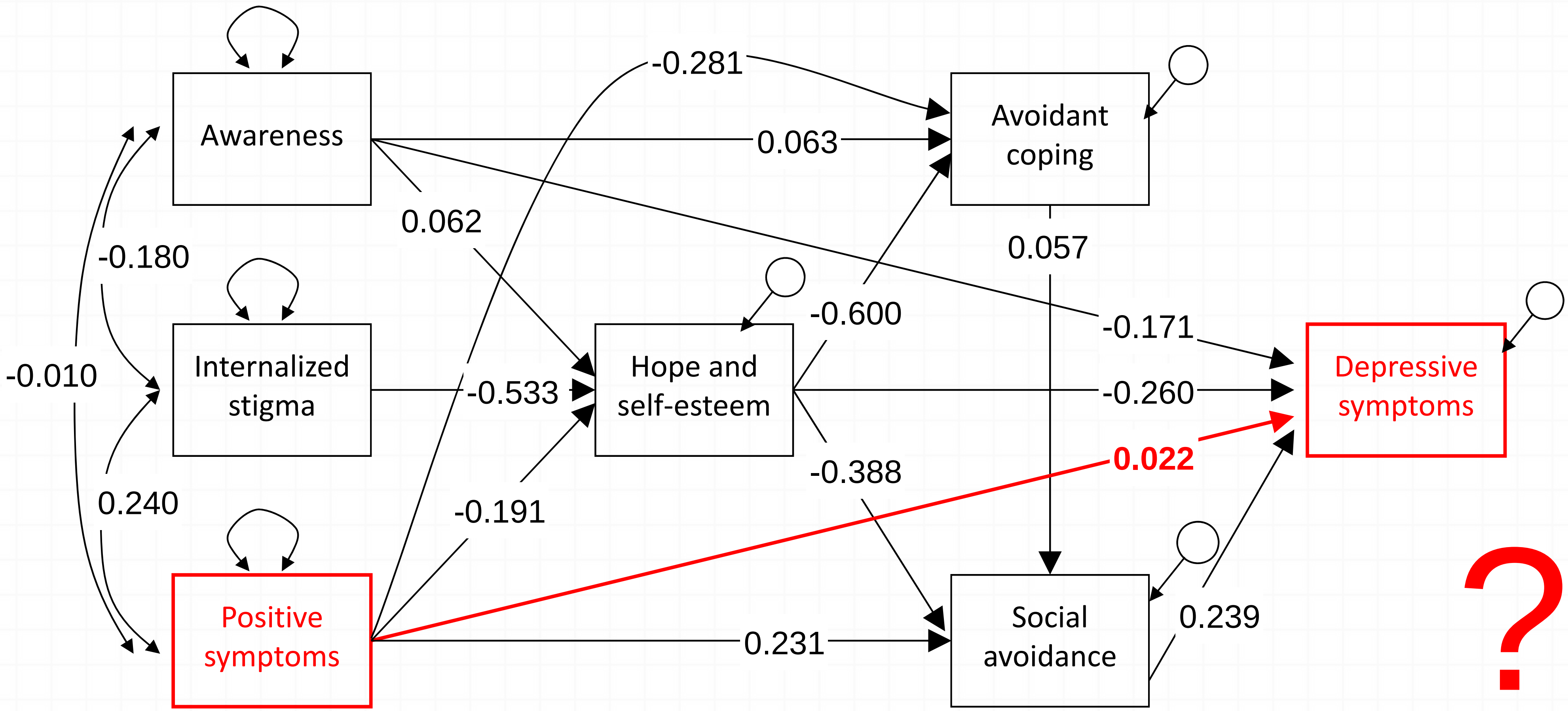
He giveth and he taketh away

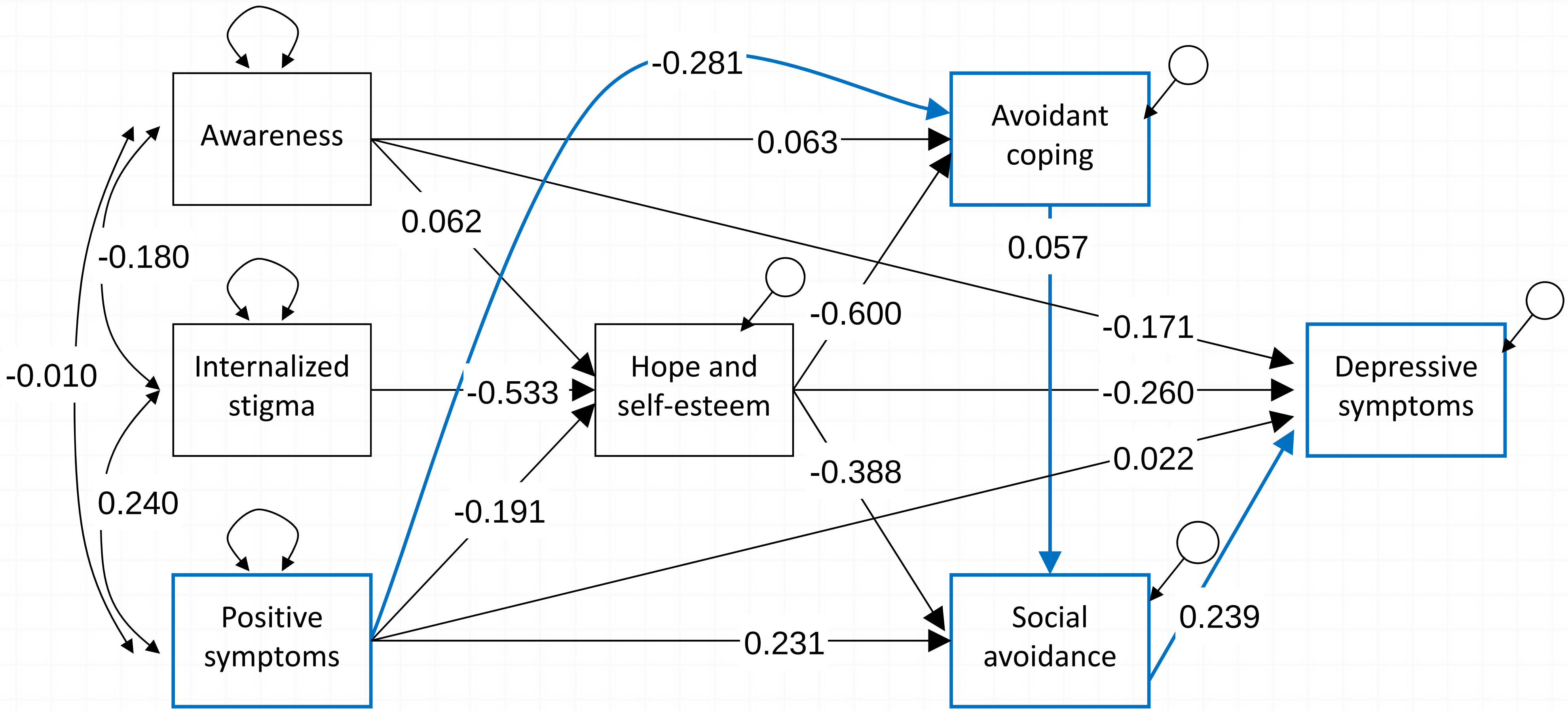
- Removing paths
- Wald / LR tests
- Could be key part of hypothesis
  - Does X affect Y?
  - Is there a direct effect of X on Y when accounting for Z?

- Adding paths
- Modification indices
- Can be abused → improve model fit

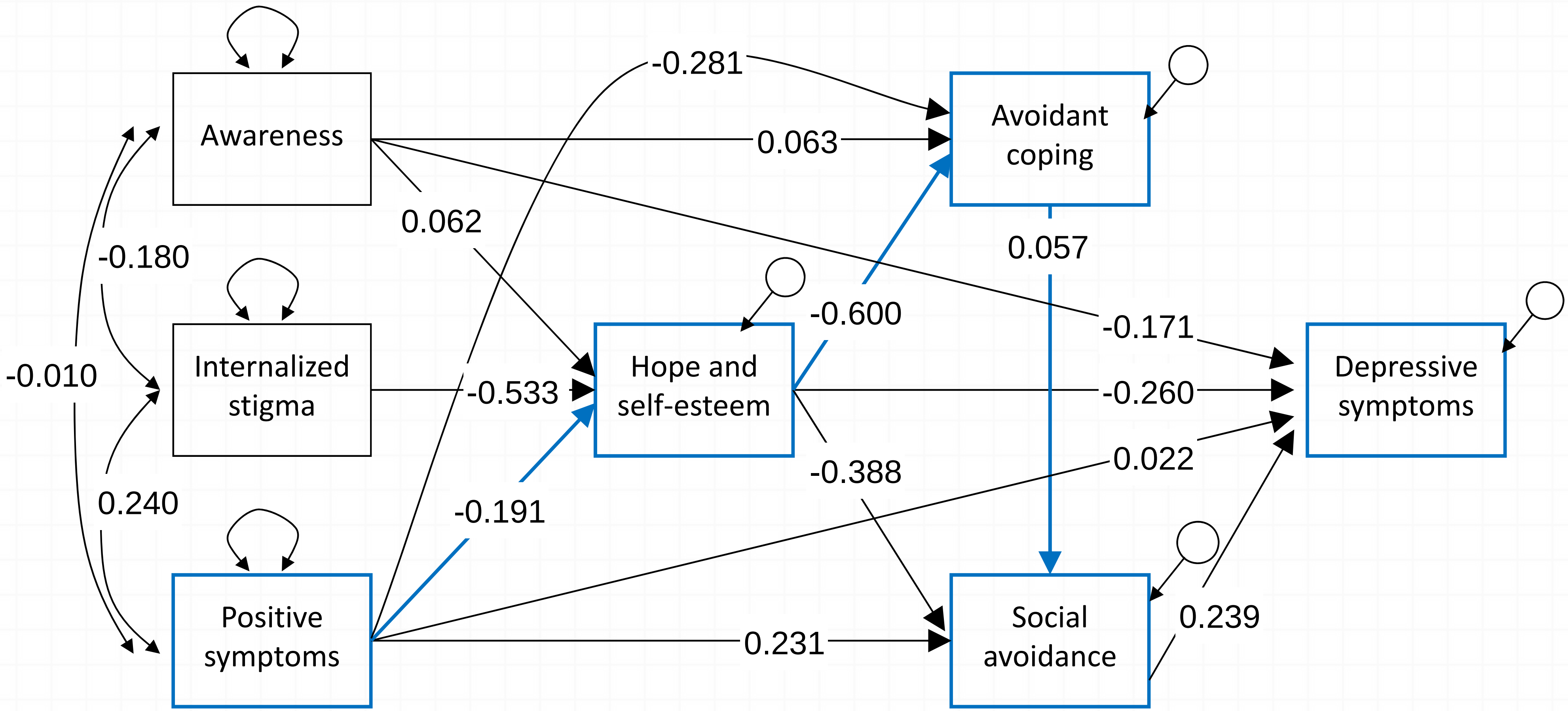
Removing paths



Indirect route 1



Indirect route 2



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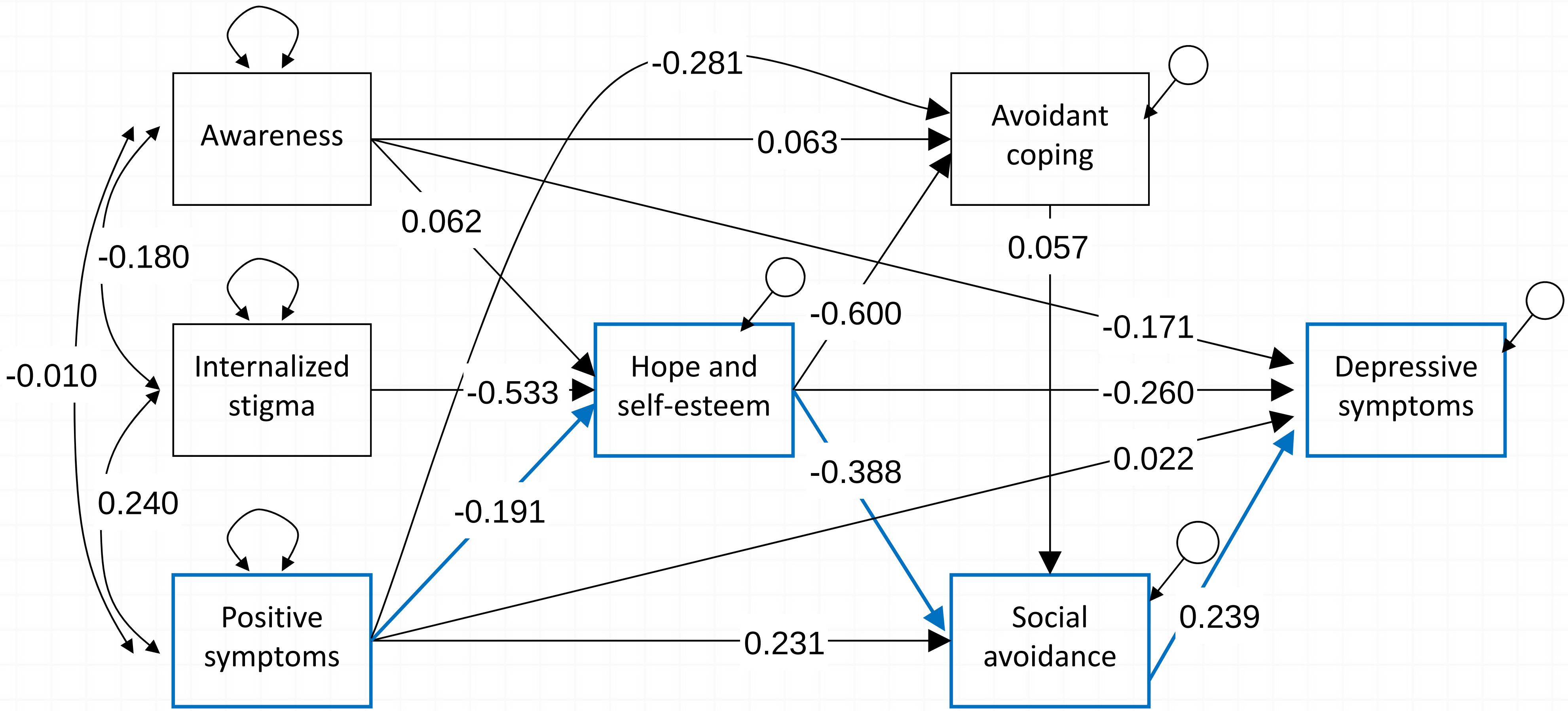
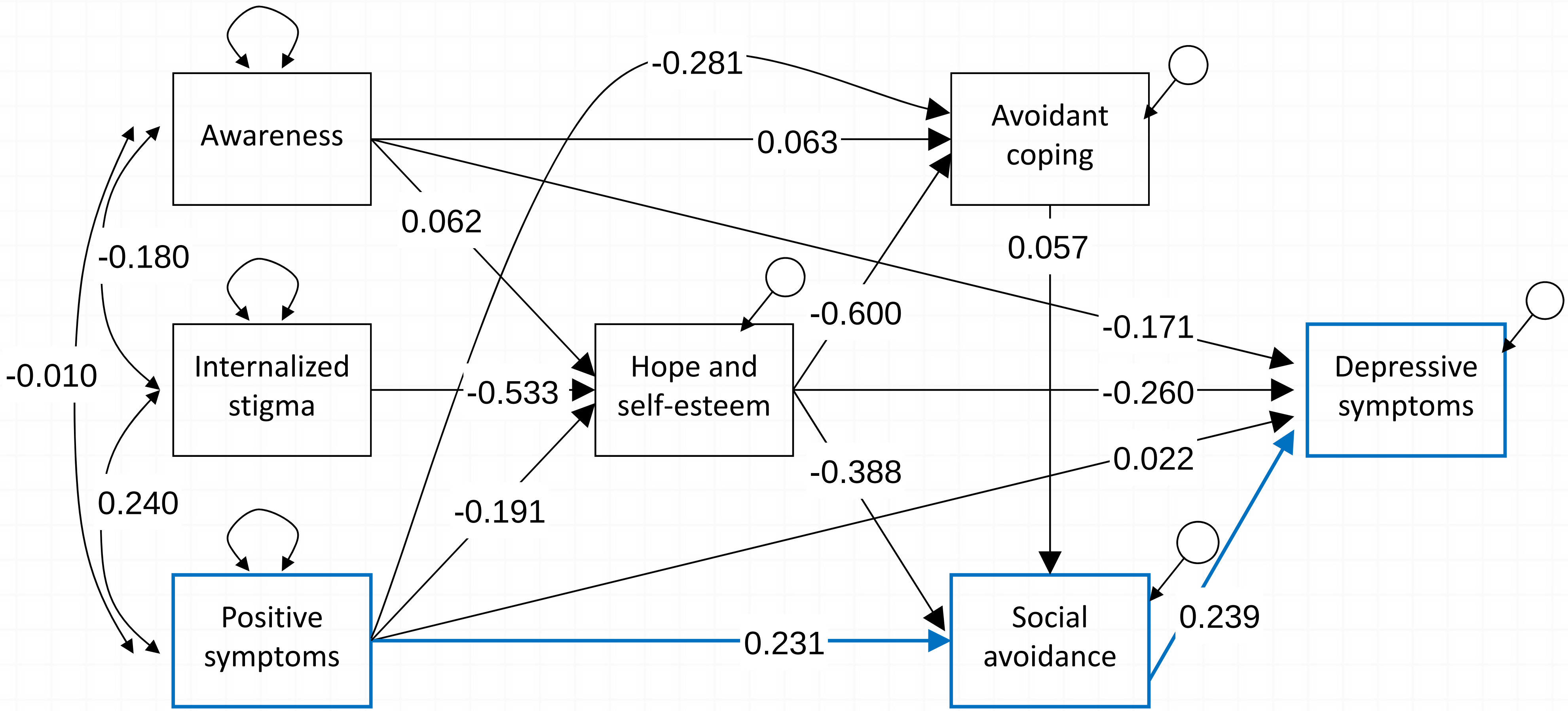
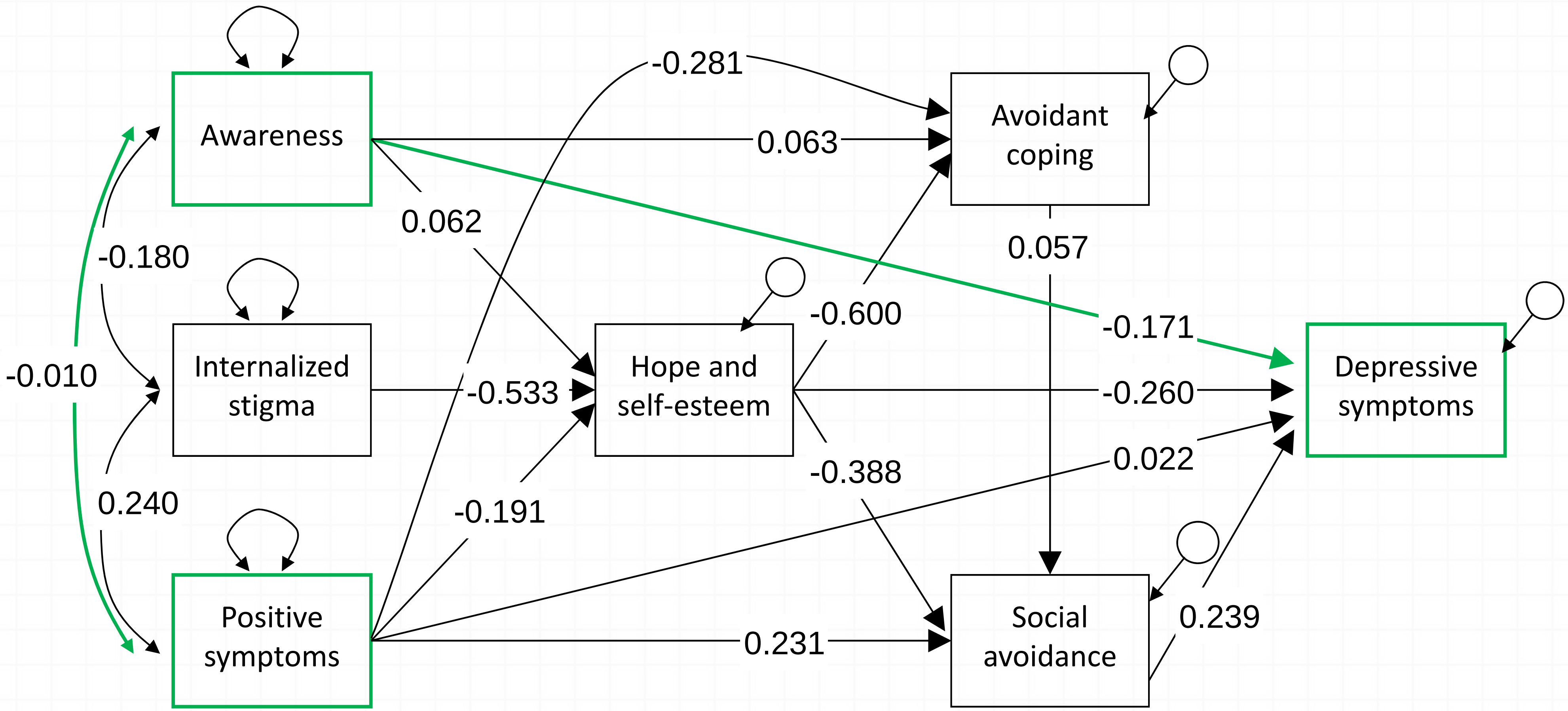
Indirect route 3



Indirect route 4



As well as by association



LR Test

```
MODEL:
    socavoid on avoidcop hope positive;
    avoidcop on aware positive hope;
    hope on aware stigma positive;
    depress on socavoid hope aware positive@0;

    hope avoidcop socavoid depress;

    aware stigma positive;
    aware with stigma positive;
    stigma with positive;
```

Unconstrained

```
MODEL FIT INFORMATION

Number of Free Parameters                    23

Loglikelihood
          H0 Value                    -1251.477
          H1 Value                    -1249.739

Chi-Square Test of Model Fit
          Value                           3.475
          Degrees of Freedom                  5
          P-Value                        0.6271
```

Constrained

```
MODEL FIT INFORMATION

Number of Free Parameters                    22

Loglikelihood
          H0 Value                    -1251.505
          H1 Value                    -1249.739

Chi-Square Test of Model Fit
          Value                           3.531
          Degrees of Freedom                  6
          P-Value                        0.7398
```

Wald Test

```
MODEL:
    socavoid on avoidcop hope positive;
    avoidcop on aware positive hope;
    hope on aware stigma positive;
    depress on socavoid hope aware;
    depress on positive (to_test);

    hope avoidcop socavoid depress;

Model test:
    to_test = 0;
```

Wald Test - results

Number of Free Parameters	23
Loglikelihood	
H0 Value	-1251.477
H1 Value	-1249.739
Chi-Square Test of Model Fit	
Value	3.475
Degrees of Freedom	5
P-Value	0.6271
Wald Test of Parameter Constraints	
Value	**0.056**
Degrees of Freedom	**1**
P-Value	**0.8129**

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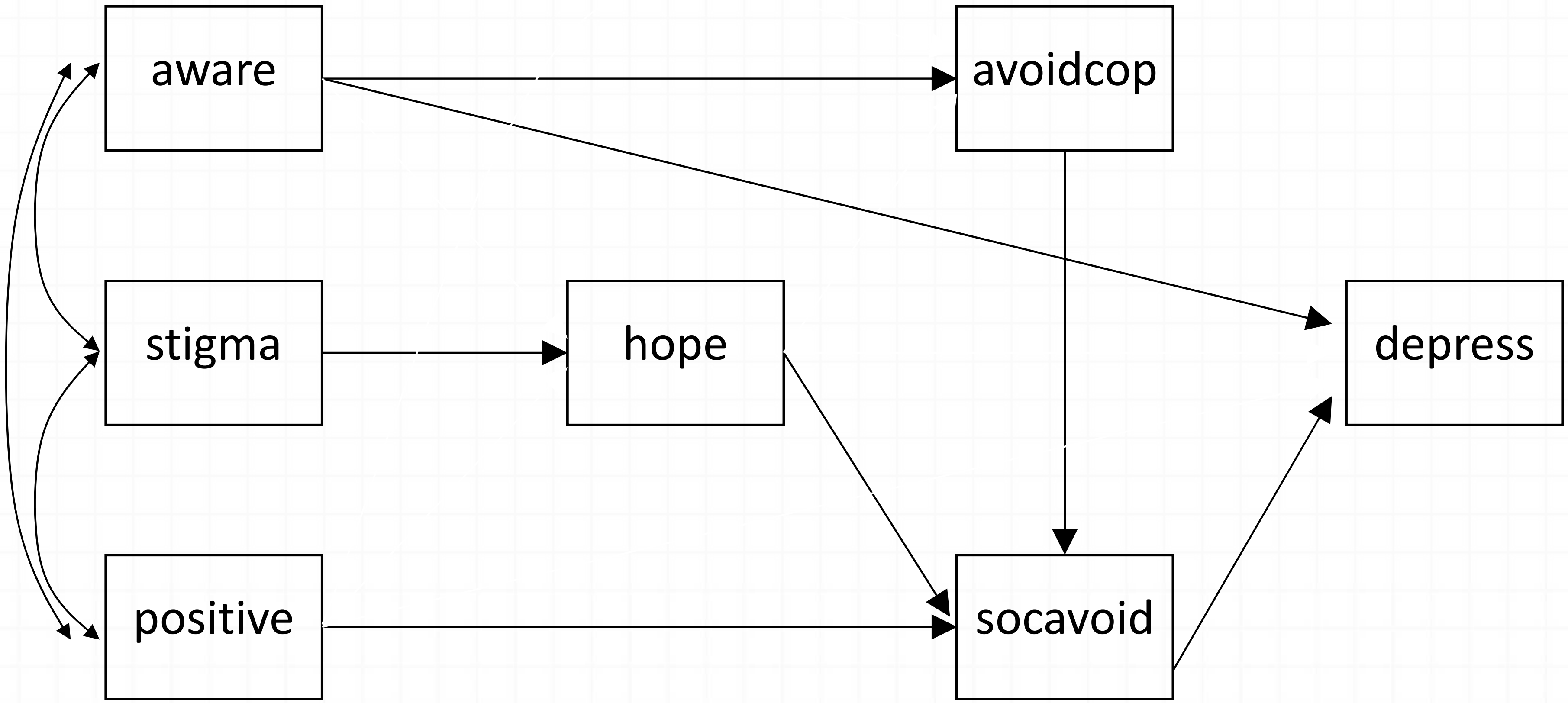
Removing paths - Summary

- Testing > 1 parameter at once
- Testing equality to other non-zero values
- Testing equality of two parameters (e.g. across groups)

- Don’t go mad!
- Stepwise / p-value approach to model refinement never a good idea

Adding paths

Start with a reduced model (otherwise no point!):-



Syntax for reduced model

```
MODEL:
    socavoid on avoidcop hope positive;
    avoidcop on aware;
    hope on stigma;
    depress on socavoid aware;

OUTPUT:
     modindices(3.8);
```

Fit is poor

```
Number of Free Parameters                         11

Chi-Square Test of Model Fit
          Value                               56.204
          Degrees of Freedom                      11
          P-Value                             0.0000

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                             0.201
          90 Percent C.I.                      0.151  0.254
          Probability RMSEA <= .05             0.000

CFI/TLI
          CFI                                  0.673
          TLI                                  0.465
```

Modindices output

```
Minimum M.I. value for printing the modification index     3.800

                                   M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.
ON Statements
HOPE      ON AVOIDCOP             20.099    -1.314      -1.314        -0.358
HOPE      ON DEPRESS               9.205    -0.295      -0.295        -0.269
HOPE      ON POSITIVE              5.284    -0.077      -0.077        -0.189
AVOIDCOP  ON HOPE                 25.321    -0.137      -0.137        -0.501
AVOIDCOP  ON SOCAVOID             12.686     0.288       0.288         0.719
AVOIDCOP  ON DEPRESS               4.493     0.068       0.068         0.227
AVOIDCOP  ON STIGMA                5.807     0.110       0.110         0.242
SOCAVOID  ON DEPRESS               3.925    -0.262      -0.262        -0.350
DEPRESS   ON HOPE                  6.192    -0.227      -0.227        -0.249

WITH Statements
AVOIDCOP WITH HOPE                19.935    -0.311      -0.311        -0.442
DEPRESS  WITH HOPE                 6.984    -0.589      -0.589        -0.276
Etc.
```

Syntax for reduced model 2

```
MODEL:
    socavoid on avoidcop hope positive;
    avoidcop on aware hope;
    hope on stigma;
    depress on socavoid aware;

OUTPUT:
     modindices(3.8);
```

Model “improvement”

```
                       M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.
ON Statements
AVOIDCOP ON HOPE      25.321    -0.137    -0.137        -0.501

First model
  Number of Free Parameters                 11
  Loglikelihood
          H0 Value                    -588.665

Revised model
  Number of Free Parameters                 12
  Loglikelihood
          H0 Value                    -573.865
```

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The other modindices have changed!

ON Statements	M.I.	E.P.C.	M.I.	E.P.C.
HOPE ON AVOIDCOP	20.099	-1.314		
HOPE ON DEPRESS	9.205	-0.295	8.747	-0.290
HOPE ON POSITIVE	5.284	-0.077	5.284	-0.077
AVOIDCOP ON HOPE	**25.321**	**-0.137**		
AVOIDCOP ON SOCAVOID	12.686	0.288	7.114	-0.380
AVOIDCOP ON DEPRESS	4.493	0.068		
AVOIDCOP ON STIGMA	5.807	0.110		
SOCAVOID ON DEPRESS	3.925	-0.262		
DEPRESS ON HOPE	6.192	-0.227	6.356	-0.233
AVOIDCOP ON POSITIVE			8.852	-0.028

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Adding paths - Summary

- Modindices can be used to indicate places where model fit can be improved
- Use with caution
- Always be led by theory

Mediation

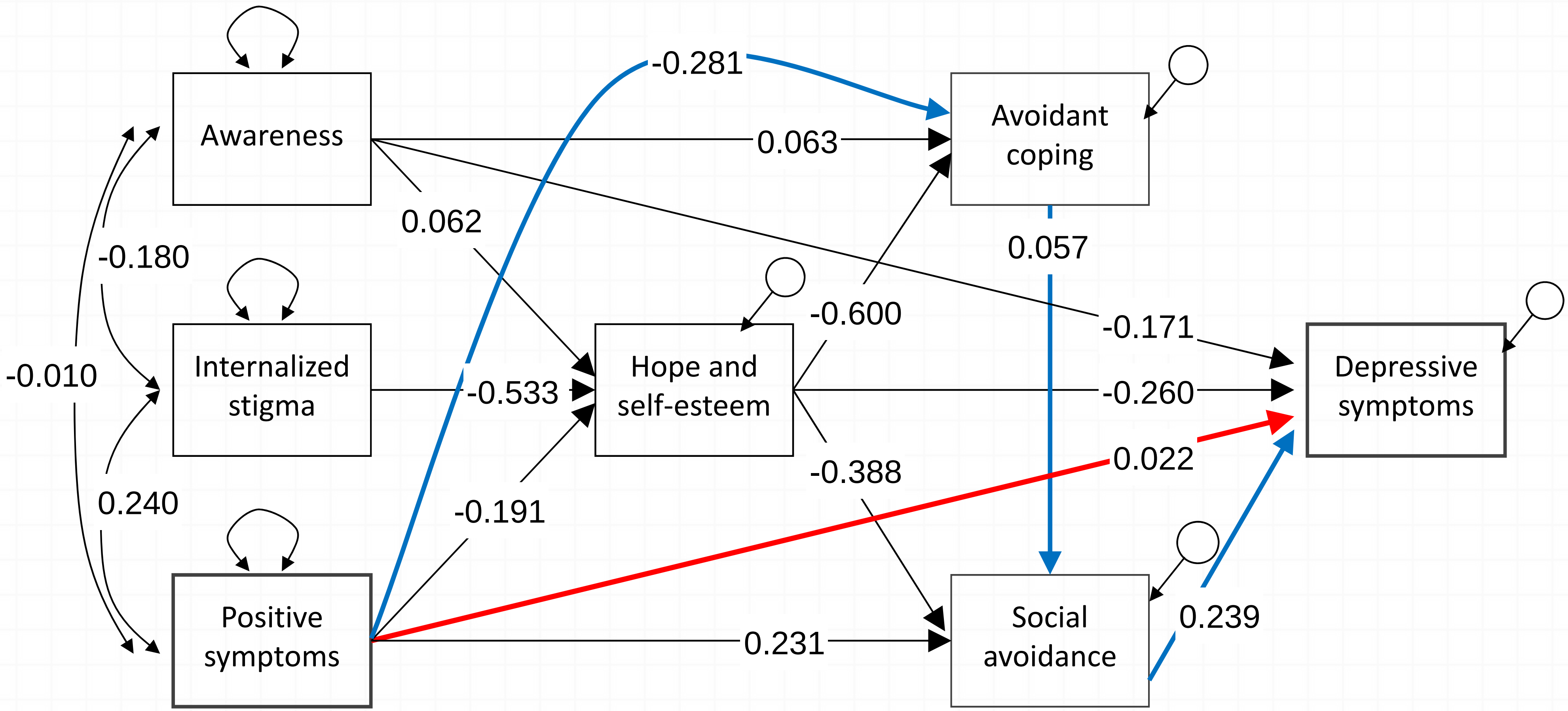
Direct and Indirect paths

What do we mean by mediation?

- Mediation in observational studies
- Mediator assumed to be part of causal sequence
- Improves our understanding

- Antenatal depression associated with child IQ
- Why might that be?
  - Parenting
  - Postnatal depression

Direct and Indirect paths



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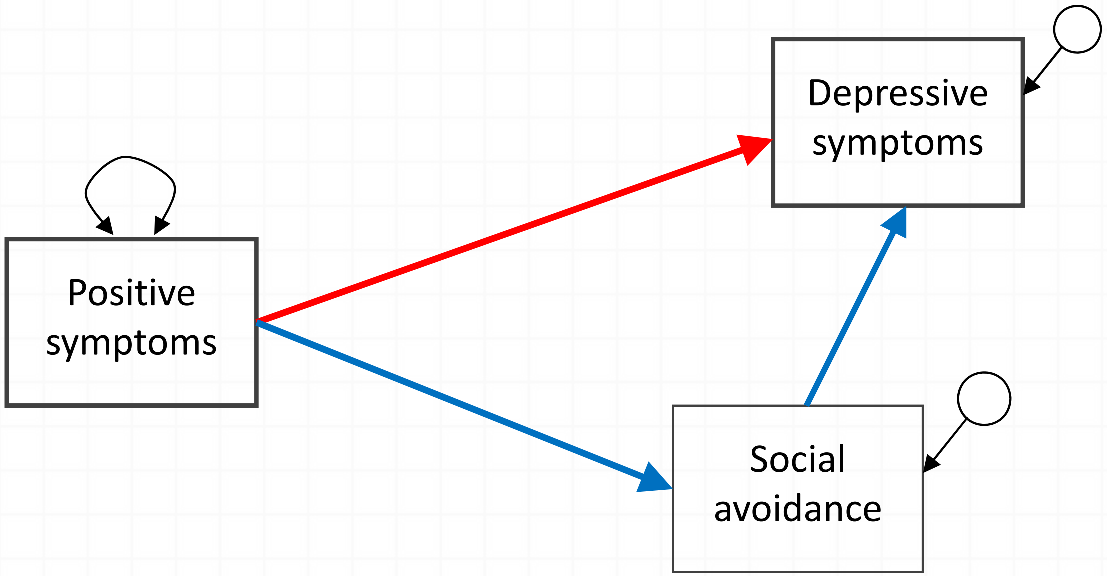
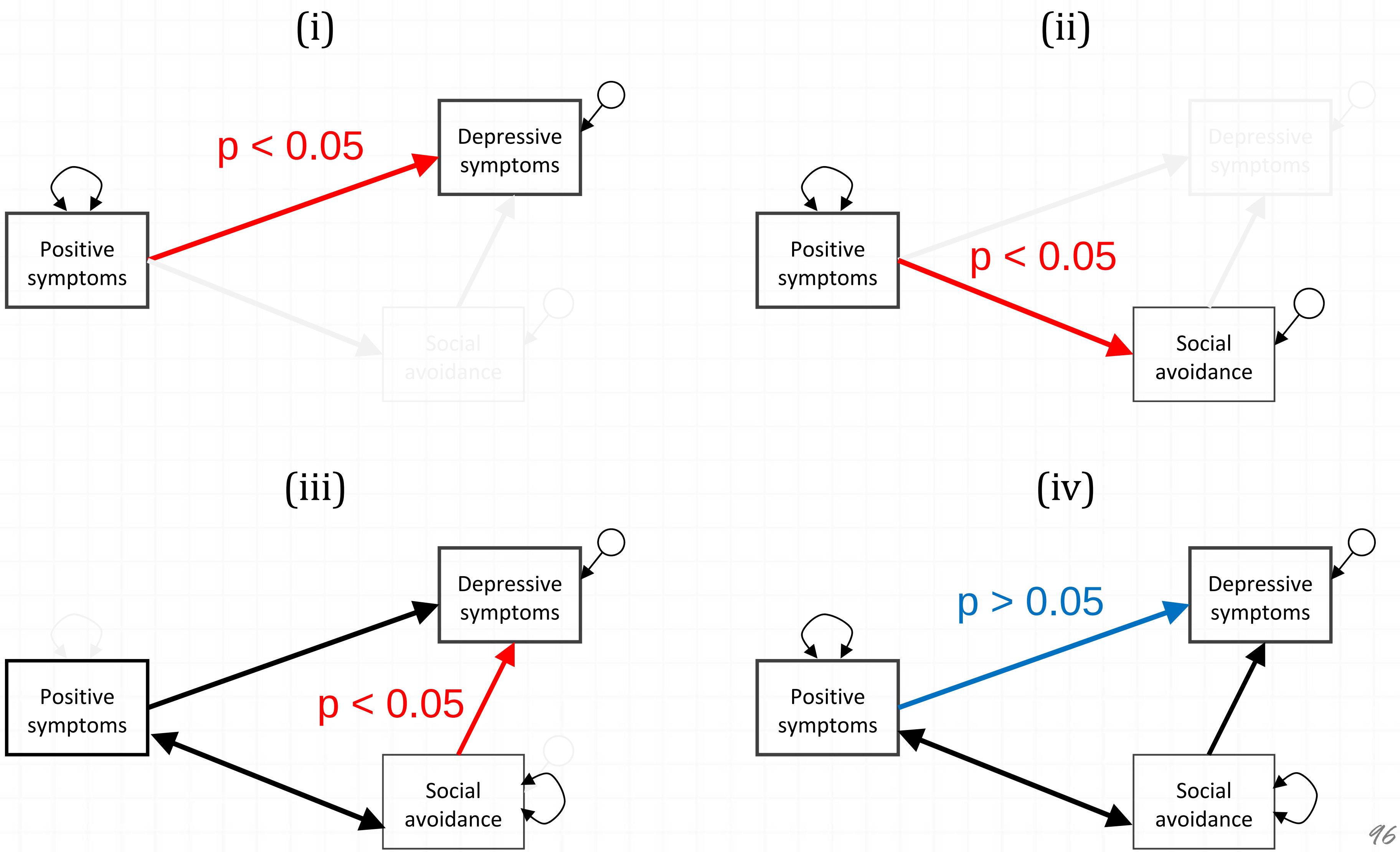
Simpler example



Baron and Kenny – causal steps



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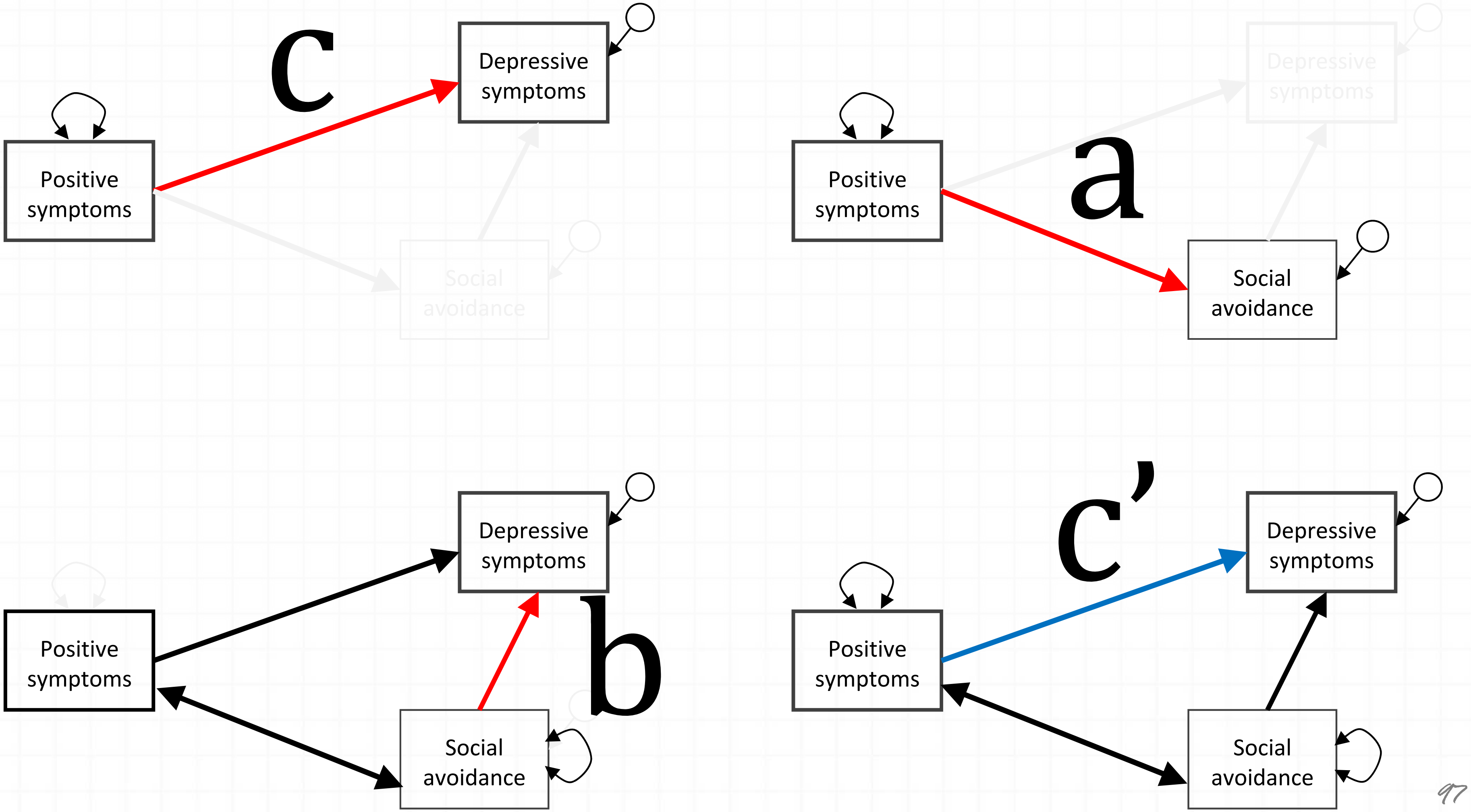
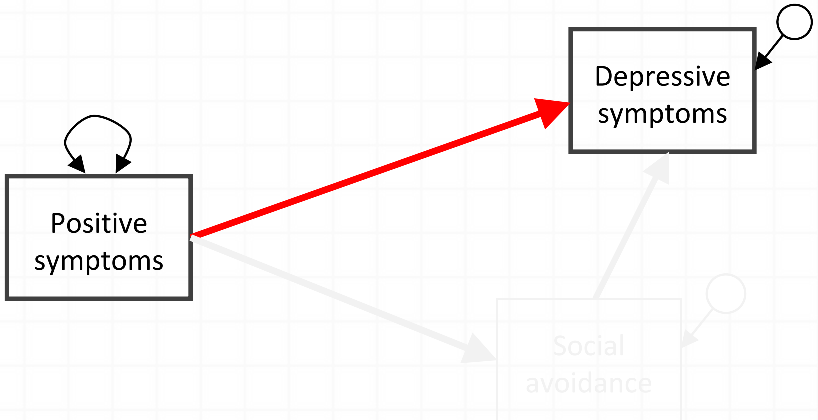
Baron and Kenny – causal steps



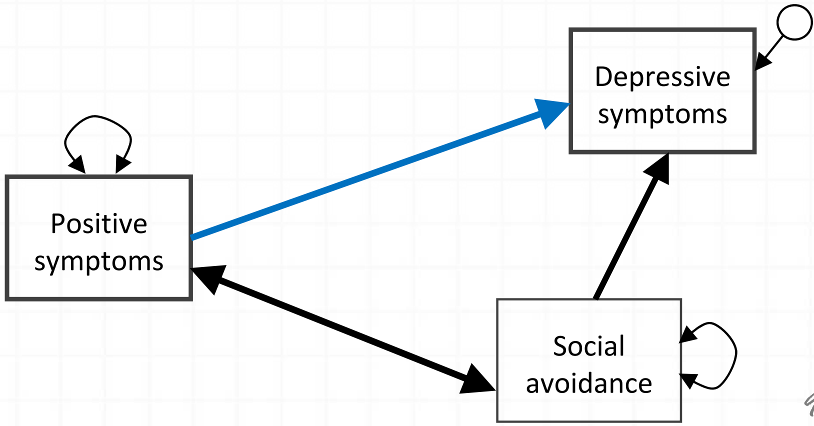
Baron and Kenny – causal steps

(i)



Total effect

(iv)



Direct effect

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Baron and Kenny – causal steps

- Very widely used
- Simple to do (e.g. In SPSS)

- Low power to detect
- Relies on p-values (from multiples tests)
- Can have mediation without a and b both being strong

- Non-significant direct-effect easier with small sample
- Should we really be rewarding small samples?

Alternative

- Directly quantify indirect effect a*b
- Sobel test: a*b/(SE(a*b)
  - OK in large samples
  - Assumes sampling distribution is normal
  - Bootstrapping favoured to derive SE's
- Evidence of non-zero indirect effect → mediation

Ratio of indirect to total effect (ab/c)

- Proportion of the total effect that is mediated
- David Mackinnon

- Can be greater than one
- Can be negative
- Gets a bit funny round c=0
- Ratio of indirect to direct – still not a proportion

In Mplus

```
VARIABLE:
  NAMES = aware stigma hope avoidcop socavoid
  depress positive;
  USEVARIABLES = socavoid depress positive;

MODEL:
    socavoid on positive;
    depress on socavoid positive;
```

Mplus results

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
SOCAVOID ON				
POSITIVE	0.099	0.026	3.773	0.000
DEPRESS ON				
SOCAVOID	0.500	0.127	3.930	0.000
POSITIVE	0.021	0.036	0.589	0.556
Residual Variances				
SOCAVOID	1.373	0.192	7.141	0.000
DEPRESS	2.270	0.318	7.141	0.000

In Mplus – Model indirect

```
VARIABLE:
  NAMES = aware stigma hope avoidcop socavoid
  depress positive;
  USEVARIABLES = socavoid depress positive;

MODEL:
    socavoid on positive;
    depress on socavoid positive;

Model indirect:
    depress IND positive;
```

Extra output obtained:-

```
TOTAL, TOTAL INDIRECT, SPECIFIC INDIRECT, AND DIRECT EFFECTS

                                                          Two-Tailed
                    Estimate       S.E.  Est./S.E.     P-Value

Effects from POSITIVE to DEPRESS
  Total                0.071      0.036      1.955       0.051
  Total indirect       0.050      0.018      2.722       0.006

  Specific indirect
    DEPRESS
    SOCAVOID
    POSITIVE           0.050      0.018      2.722       0.006

  Direct
    DEPRESS
    POSITIVE           0.021      0.036      0.589       0.556
```

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Extra output:-

```
TOTAL, TOTAL INDIRECT, SPECIFIC INDIRECT, AND DIRECT EFFECTS

                                                          Two-Tailed
                    Estimate       S.E.  Est./S.E.        P-Value

Effects from POSITIVE to DEPRESS
  Total                0.071      0.036      1.955          0.051
  Total indirect       0.050      0.018      2.722          0.006

  Specific indirect
    DEPRESS
    SOCAVOID
    POSITIVE           0.050      0.018      2.722          0.006

  Direct
    DEPRESS
    POSITIVE           0.021      0.036      0.589          0.556
```

Route taken

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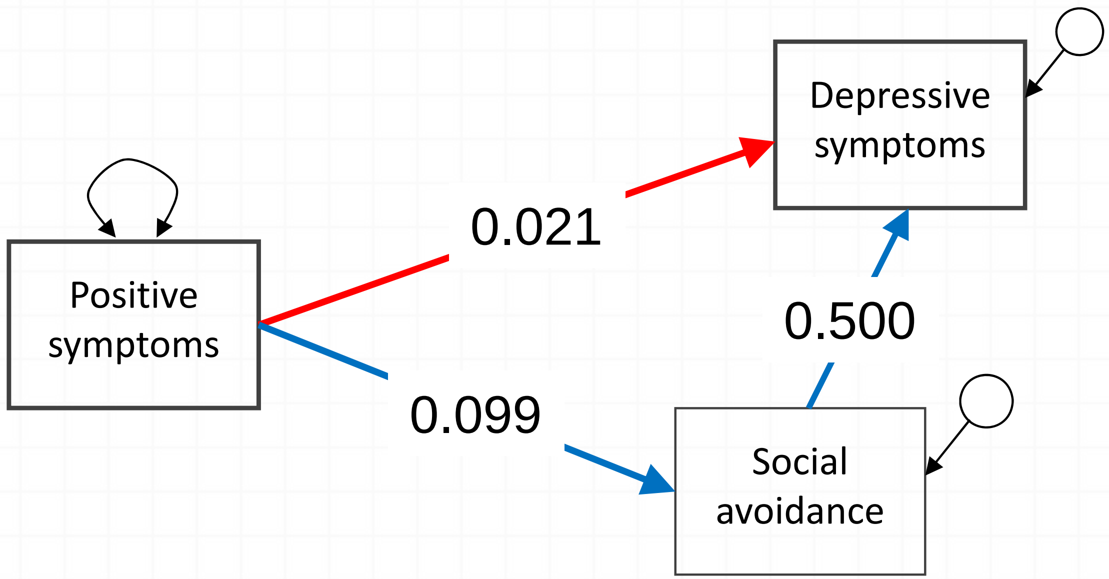
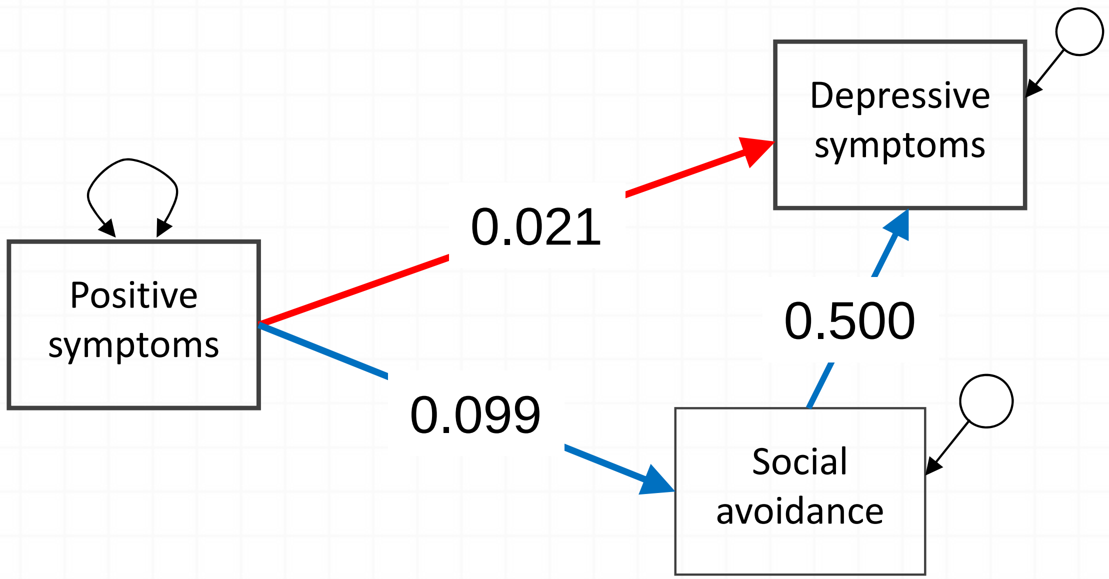
```
SOCAVOID ON
   POSITIVE        0.099 (0.026)

DEPRESS  ON
   SOCAVOID        0.500 (0.127)
   POSITIVE        0.021 (0.036)

Residual Variances
   SOCAVOID        1.373 (0.192)
   DEPRESS         2.270 (0.318)
```



```
SOCAVOID  ON
   POSITIVE        0.099 (0.026)

DEPRESS   ON
   SOCAVOID        0.500 (0.127)
   POSITIVE        0.021 (0.036)

Residual Variances
   SOCAVOID        1.373 (0.192)
   DEPRESS         2.270 (0.318)
```




```
Effects from POSITIVE to DEPRESS
   Total                  0.071 (0.036)
   Total indirect         0.050 (0.018)
   Direct                 0.021 (0.036)
```

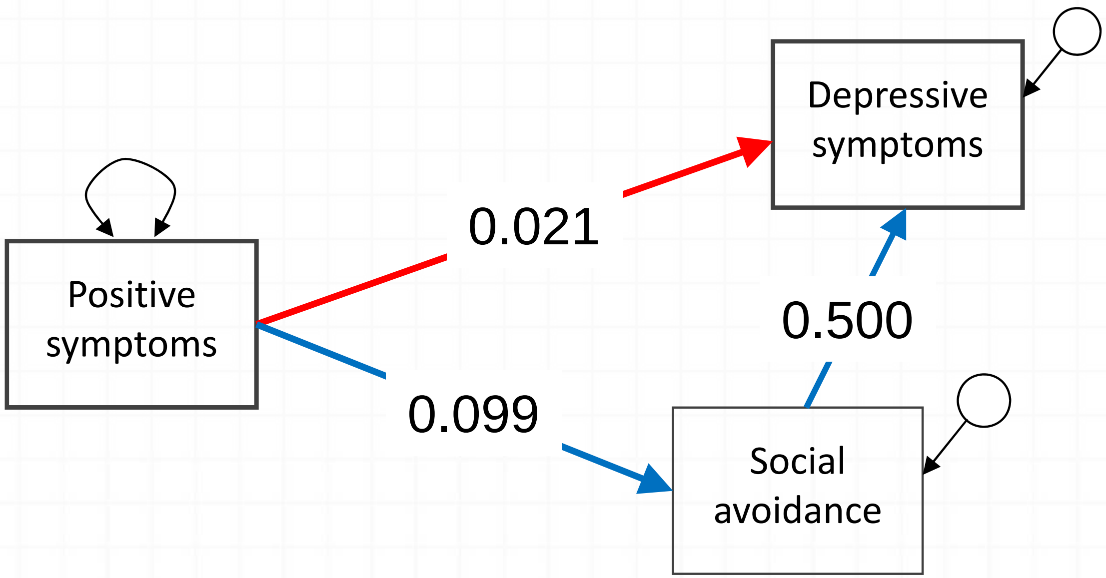
Indirect Effect

= product of paths

= 0.099 * 0.500

So how do we interpret this then?

```
Effects from POSITIVE to DEPRESS
   Total                   0.071 (0.036)
   Total indirect          0.050 (0.018)
   Direct                  0.021 (0.036)
```

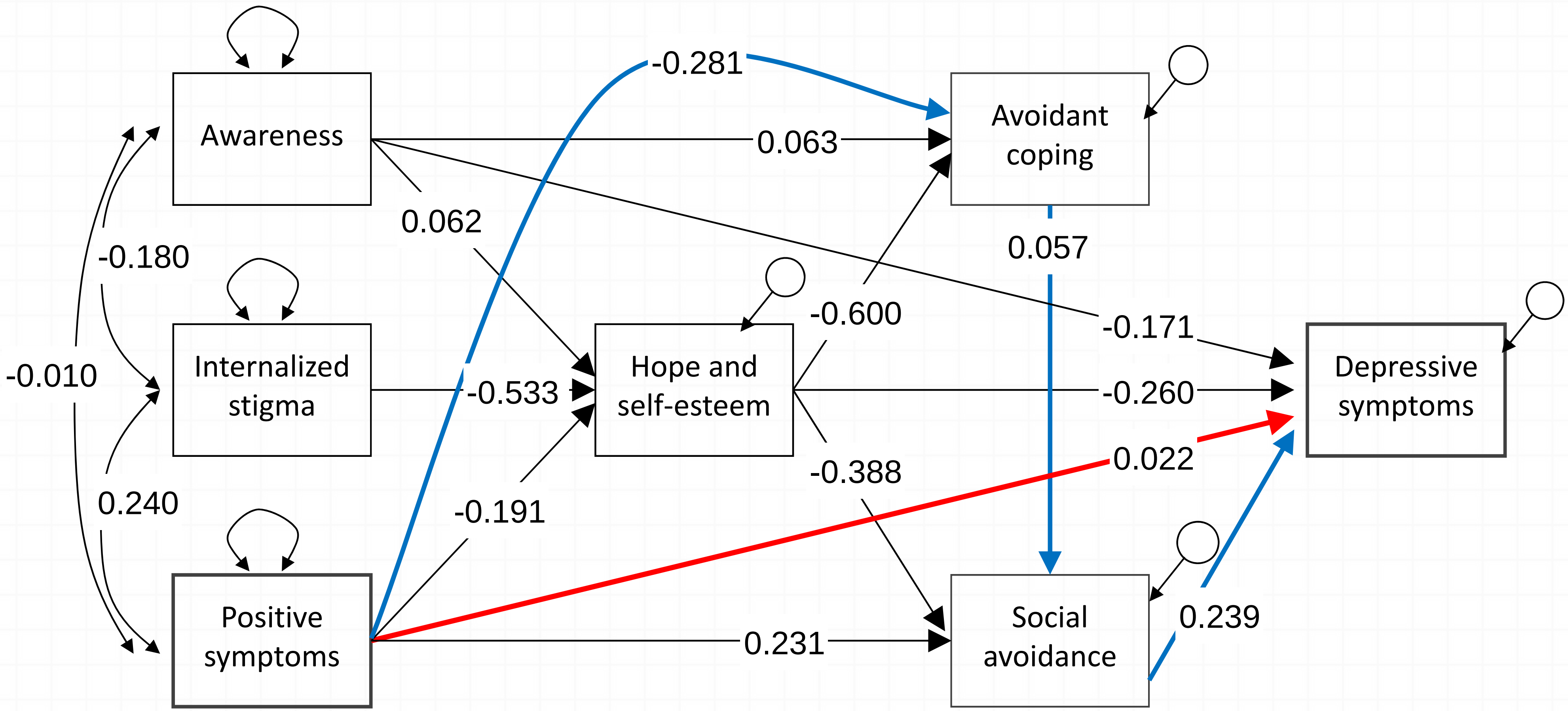


Strong evidence of a non-zero indirect effect

Substantial part of total effect of positive symptoms on depression is mediated through social avoidance (given the current model)

Take a deep breath!

Now for a more complex example



depress IND positive;

```
Effects from POSITIVE to DEPRESS
  Total                                                     0.053 (0.035)
  Total indirect                                            0.045 (0.017)

 Specific indirect
  POSITIVE → HOPE → DEPRESS                                 0.019 (0.011)
  POSITIVE → SOCAVOID → DEPRESS                             0.021 (0.012)
  POSITIVE → HOPE → SOCAVOID → DEPRESS                      0.007 (0.004)
  POSITIVE → AVOIDCOP → SOCAVOID → DEPRESS                 -0.001 (0.003)
  POSITIVE → HOPE → AVOIDCOP → SOCAVOID → DEPRESS           0.531 (0.595)

 Direct
  POSITIVE → DEPRESS                                        0.008 (0.035)
```

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Positive to Depress VIA Hope

- Model indirect:
  - depress VIA hope positive;

```
Effects from POSITIVE to DEPRESS via HOPE

 Sum of indirect                                        0.026 (0.013)

 Specific indirect
  POSITIVE → HOPE → DEPRESS                             0.019 (0.011)
  POSITIVE → HOPE → SOCAVOID → DEPRESS                  0.007 (0.004)
  POSITIVE → HOPE → AVOIDCOP → SOCAVOID → DEPRESS       0.531 (0.595)
```

Summary – direct/indirect effects

- IND and VIA
  - provides information on direct/indirect pathways
  - Ideally should be used with bootstrapping
- Model dependent
  - Direct effect will diminish with model complexity
- Mediation
  - Extent to which a total effect is partitioned into indirect and direct components

Mediation models 2

Including binary measures

Binary data in mediation models

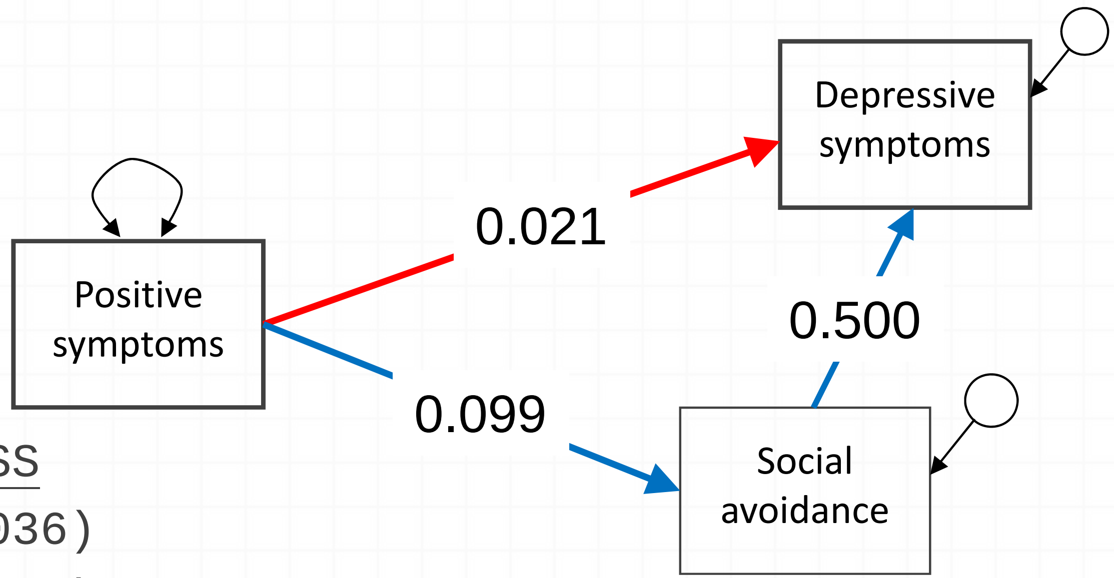
- As a mediator / intermediate variable
- As an outcome
- As an exogenous variable
  - Makes no difference
  - Categorical treated as continuous (dummies)

With continuous data

```
SOCAVOID  ON
   POSITIVE       0.099 (0.026)

DEPRESS   ON
   SOCAVOID       0.500 (0.127)
   POSITIVE       0.021 (0.036)

Effects from POSITIVE to DEPRESS
   Total                0.071 (0.036)
   Total indirect       0.050 (0.018)
   Direct               0.021 (0.036)
```



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With a continuous outcome Y

- Variance of outcome Y is known
- Fixed across models with different covariates
- Ordinary regression models have a fixed scale

- Can fit a number of regression models
  - Indirect/mediated effect = total effect – direct effect = c-c'
- Or can fit a single SEM model
  - Indirect/mediated effect = product of paths = a*b

With a binary outcome Y

- Unobserved continuous variable Y* underlies binary Y
- Variance of Y* is unknown
- Residual variance for logit/probit models fixed (1, $\pi^2/3$)
- Scale depends on variables in the model

- Regression approach (c-c')
  - Misleading results
  - Rescaling is possible
- SEM approach with categorical option still valid

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Parameter rescaling – quick comment

- Parameters from separate regression not comparable
- Multiply each coefficient by the SD of the predictor variable in the equation and then dividing by the SD of the outcome variable.
- Excel spreadsheet
  - http://nrherr.bol.ucla.edu/Mediation/logmed.html
- Stata function "binary_mediation" does the same thing
  - And allows bootstrapping to be incorporated

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Mplus – probit & logit with a binary Y

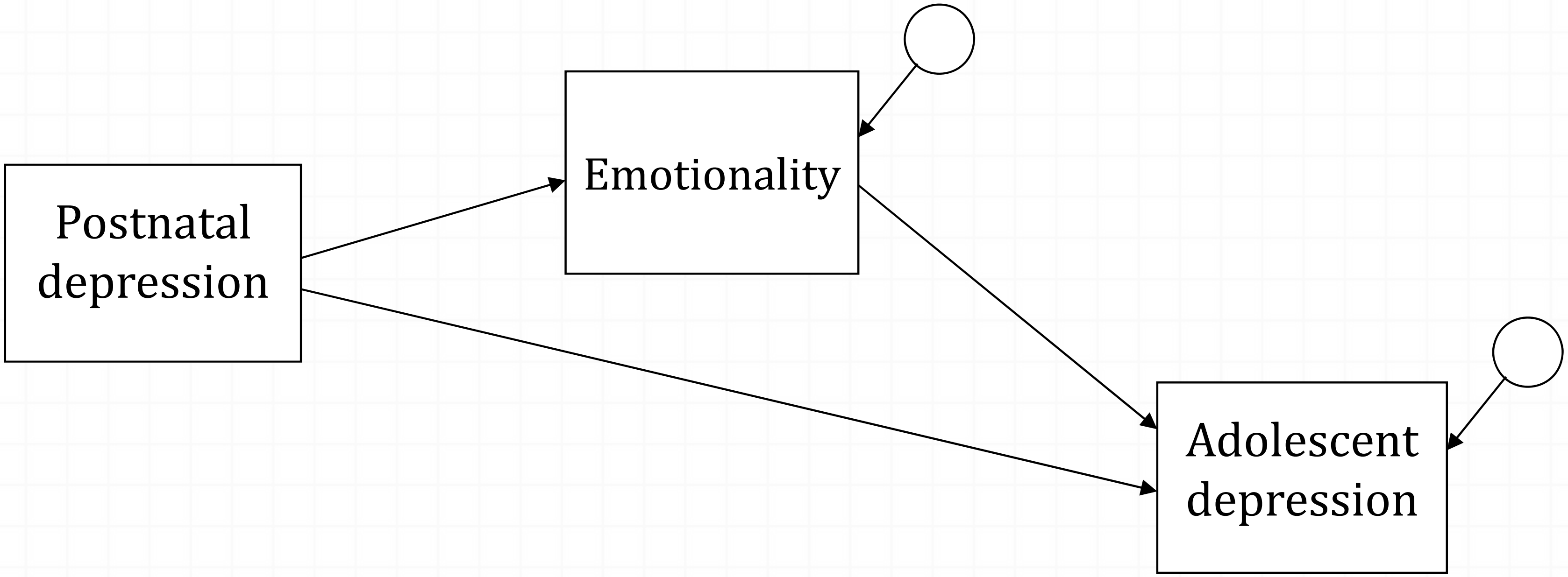
- ML (logit/probit)
  - Y is modelled as Y* when Y is the dependent variable
  - Y is modelled as Y when Y is the independent variable
- WLSMV (probit)
  - Y is modelled as Y* when Y is the dependent variable
  - Y is modelled as Y* when Y is the independent variable

So what does that mean?

- In standard binary outcome regression, logit and probit models are roughly equivalent
- In SEM mediation models conclusions may differ depending on method and estimator used

- Effect of binary M on outcome Y will not be comparable across modelling approaches
- Irrespective of whether Y is continuous or binary

A logit/probit example



Postnatal depression (mdep_pn) is binary, treated as continuous
Emotionality (emo_bin) is binary and treated as such
Adolescent depression (mfqsum18) is continuous

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Probit model - WLSMV

```
Define:
      emo_bin = (emototT3 >10);
      mfqsum18 = mfq18_01 + mfq18_02 + mfq18_03 + ...+ mfq18_13;
Variable:
     Usevariables = mdep_pn emo_bin mfqsum18;
     Categorical = emo_bin;
Analysis:
     estimator = WLSMV;
Model:
     mfqsum18 on mdep_pn emo_bin;
     emo_bin on mdep_pn;
Model indirect:
     mfqsum18 IND mdep_pn;
```

Probit model - WLSMV

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
MFQSUM18 ON				
MDEP_PN	0.988	0.339	2.911	0.004
EMO_BIN	0.551	0.186	2.959	0.003
EMO_BIN ON				
MDEP_PN	0.666	0.090	7.386	0.000

TOTAL, TOTAL INDIRECT, SPECIFIC INDIRECT, AND DIRECT EFFECTS

Effects from MDEP_PN to MFQSUM18				
Total	1.355	0.318	4.255	0.000
Specific indirect	0.367	0.133	2.757	0.006
Direct	0.988	0.339	2.911	0.004

Probit model - ML

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
MFQSUM18 ON				
MDEP_PN	**1.145**	**0.341**	**3.358**	**0.001**
EMO_BIN	**1.100**	**0.361**	**3.048**	**0.002**
EMO_BIN ON				
MDEP_PN	**0.666**	**0.090**	**7.386**	**0.000**

Logit model - ML

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
MFQSUM18 ON				
MDEP_PN	1.145	0.341	3.358	0.001
EMO_BIN	1.100	0.361	3.048	0.002
EMO_BIN ON				
MDEP_PN	1.162	0.154	7.548	0.000

LOGISTIC REGRESSION ODDS RATIO RESULTS

EMO_BIN ON	
MDEP_PN	3.195

Scaled parameters approach (e.g. Stata)

Logit: emo_bin on iv (a1 path)

emo_bin	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
mdep_pn	1.161604	.1539016	7.55	0.000	.859962	1.463245
_cons	-1.924484	.0859839	-22.38	0.000	-2.093009	-1.755959

OLS regression: dv on iv (c path)

mfqtot18	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
mdep_pn	1.355169	.335292	4.04	0.000	.697477	2.01286
_cons	6.047658	.1456367	41.53	0.000	5.761985	6.333332

OLS regression: dv on mv & iv (b & c' paths)

mfqtot18	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
emo_bin	1.100295	.3613163	3.05	0.002	.3915547	1.809035
mdep_pn	1.145388	.3413925	3.36	0.001	.4757294	1.815046
_cons	5.907522	.1523523	38.78	0.000	5.608675	6.206369

Binary Mediation summary

- With probit/WLSMV the indirect effect can be directly outputted using "model indirect"
- However this yields main effects that are more difficult to interpret (not like odds ratios)

- Output using ML is not scaled so path estimates cannot simply be multiplied to provide estimate of indirect effect
- Re-scaling should be possible to get best of both worlds and yield results that agree with Stata – watch this space...

Further mediation reading

- Andrew F. Hayes (2009): Beyond Baron and Kenny: Statistical Mediation Analysis in the New Millennium, Communication Monographs, 76:4, 408-420.
- Mackinnon, David Peter. Introduction to statistical mediation analysis. Lawrence Erlbaum and Associates (2008).
- David P. Mackinnon, Lockwood, C. M., Brown, C. H., Wang, W. & Hoffman, J. M.. The intermediate endpoint effect in logistic and probit regression. Clinical Trials (2007).
- http://nrherr.bol.ucla.edu/Mediation/logmed.html
- Also see "binary_mediation" Stata command